Continuous Function.

(1) **A list of continuous functions:**

Function $f(x)$	**Interval in which $f(x)$ is continuous**
(i) Constant K	$(-\infty, \infty)$
(ii) x^n, (n is a positive integer)	$(-\infty, \infty)$
(iii) x^{-n} (n is a positive integer)	$(-\infty, \infty) - \{0\}$
(iv) $\lvert x - a \rvert$	$(-\infty, \infty)$
(v) $p(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots\dots + a_n$	$(-\infty, \infty)$
(vi) $\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial in x	$(-\infty, \infty) - \{x : q(x) = 0\}$
(vii) $\sin x$	$(-\infty, \infty)$
(viii) $\cos x$	$(-\infty, \infty)$
(ix) $\tan x$	$(-\infty, \infty) - \{(2n + 1)\pi/2 : n \in I\}$
(x) $\cot x$	$(-\infty, \infty) - \{n\pi : n \in I\}$
(xi) $\sec x$	$(-\infty, \infty) - \{(2n + 1)\pi/2 : n \in I\}$
(xii) $\text{cosec } x$	$(-\infty, \infty) - \{n\pi : n \in I\}$
(xiii) e^x	$(-\infty, \infty)$
(xiv) $\log_e x$	$(0, \infty)$

(2) **Properties of continuous functions:** Let $f(x)$ and $g(x)$ be two continuous functions at $x = a$. Then

(i) $cf(x)$ is continuous at $x = a$, where c is any constant

(ii) $f(x) \pm g(x)$ is continuous at $x = a$.

(iii) $f(x) \,.\, g(x)$ is continuous at $x = a$.

(iv) $f(x)/g(x)$ is continuous at $x = a$, provided $g(a) \neq 0$.

Important Tips

- ☞ *A function* $f(x)$ *is said to be continuous if it is continuous at each point of its domain.*
- ☞ *A function* $f(x)$ *is said to be everywhere continuous if it is continuous on the entire real line R i.e.* $(-\infty, \infty)$ *. Eg. Polynomial function* e^x, $\sin x, \cos x$, *constant,* $x^n, |x-a|$ *etc.*
- ☞ *Integral function of a continuous function is a continuous function.*
- ☞ *If g(x) is continuous at x = a and f(x) is continuous at x = g (a) then (fog) (x) is continuous at* $x = a$ *.*
- ☞ *If f(x) is continuous in a closed interval [a, b] then it is bounded on this interval.*
- ☞ *If f(x) is a continuous function defined on [a, b] such that f (a) and f (b) are of opposite signs, then there is atleast one value of x for which f(x) vanishes. i.e. if f (a)>0, f (b) < 0* $\Rightarrow \exists c \in (a, b)$ *such that f(c) = 0.*
- ☞ *If f(x) is continuous on [a, b] and maps [a, b] into [a, b] then for some x ∈ [a, b] we have f(x) = x.*

(3) **Continuity of composite function:** If the function $u = f(x)$ is continuous at the point $x = a$, and the function $y = g(u)$ is continuous at the point $u = f(a)$, then the composite function $y = (gof)(x) = g(f(x))$ is continuous at the point *x* = *a*.