

Discontinuous Function.

(1) **Discontinuous function:** A function ' f ' which is not continuous at a point $x = a$ in its domain is said to be discontinuous there at. The point ' a ' is called a point of discontinuity of the function.

The discontinuity may arise due to any of the following situations.

(i) $\lim_{x \rightarrow a^+} f(x)$ or $\lim_{x \rightarrow a^-} f(x)$ or both may not exist

(ii) $\lim_{x \rightarrow a^+} f(x)$ as well as $\lim_{x \rightarrow a^-} f(x)$ may exist, but are unequal.

(iii) $\lim_{x \rightarrow a^+} f(x)$ as well as $\lim_{x \rightarrow a^-} f(x)$ both may exist, but either of the two or both may not be equal to $f(a)$.

Important Tips

☞ A function f is said to have removable discontinuity at $x = a$ if $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$ but their common value is not equal to $f(a)$.

Such a discontinuity can be removed by assigning a suitable value to the function f at $x = a$.

☞ If $\lim_{x \rightarrow a} f(x)$ does not exist, then we cannot remove this discontinuity. So this become a non-removable discontinuity or essential discontinuity.

☞ If f is continuous at $x = c$ and g is discontinuous at $x = c$, then

(a) $f + g$ and $f - g$ are discontinuous (b) $f \cdot g$ may be continuous

☞ If f and g are discontinuous at $x = c$, then $f + g$, $f - g$ and fg may still be continuous.

☞ Point functions (domain and range consists one value only) is not a continuous function.
