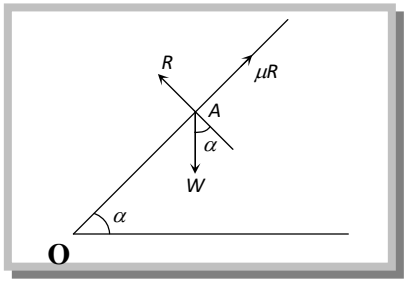
# Limiting equilibrium on an Inclined Plane.

Let a body of weight W be on the point of sliding down a plane which is inclined at an angle $\alpha$ to the horizon. Let R be the normal reaction and μ R be the limiting friction acting up the plane.




Thus, the body is in limiting equilibrium under the action of three forces: R, μ R and W.

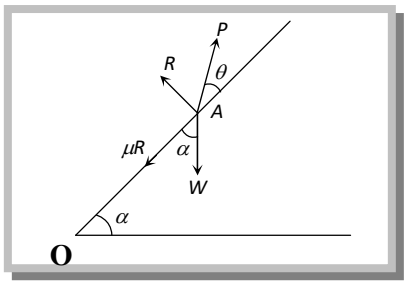
Resolving the forces along and perpendicular to the plane, we have

$\mu R = W \sin\alpha$ and $R = W\cos\alpha$

$$\Rightarrow \frac{\mu R}{R} = \frac{W\sin\alpha}{\cos\alpha} \Rightarrow \mu = \tan\alpha \Rightarrow \tan\lambda = \tan\alpha \Rightarrow \alpha = \lambda$$

Thus, if a body be on the point of sliding down an inclined plane under its own weight, the inclination of the plane is equal to the angle of the friction.

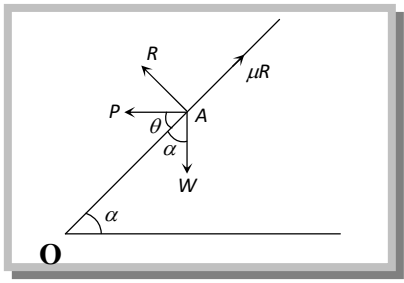
(1) **Least force required to pull a body up an inclined rough plane:** Let a body of weight W be at point A, $\alpha$ be the inclination of rough inclined plane to the horizontal and $\lambda$ be the angle of friction. Let P be the force acting at an angle $\theta$ with the plane required just to move body up the plane.




$$P = W\frac{\sin(\alpha+\lambda)}{\cos(\theta-\lambda)} \qquad \{\because \mu = \tan\lambda\}$$

Clearly, the force P is least when $\cos(\theta-\lambda)$ is maximum, i.e. when $\cos(\theta-\lambda)=1$, i.e. $\theta-\lambda=0$ or $\theta=\lambda$. The least value of P is $W\sin(\alpha+\lambda)$

(2) **Least force required to pull a body down an inclined plane:** Let a body of weight W be at the point A, $\alpha$ be the inclination of rough inclined plane to the horizontal and $\lambda$ be the angle of friction. Let P be the force acting an angle $\theta$ with the plane, required just to move the body up the plane.




$$P = \frac{W\sin(\lambda-\alpha)}{\cos(\theta-\lambda)} \qquad [\because \mu = \tan\lambda]$$

Clearly, P is least when $\cos(\theta-\lambda)$ is maximum, i.e. when $\theta-\lambda=0$ or $\theta=\lambda$.

The least value of P is $W\sin(\lambda-\alpha)$.

Note: If $\alpha=\lambda$, then the body is in limiting equilibrium and is just on the point of moving downwards.

If $\alpha<\lambda$, then the least force required to move the body down the plane is $W\sin(\lambda-\alpha)$.

If $\alpha=\lambda, \alpha>\lambda$ or $\alpha<\lambda$, then the least force required to move the body up the plane is $W\sin(\alpha+\lambda)$.

❑ If $\alpha > \lambda$, then the body will move down the plane under the action of its weight and normal reaction.

**Important Tips**

☞ **Least force on the horizontal plane :** Least force required to move the body with weight W on the rough horizontal plane is W sin $\lambda$ .