Centre of Gravity.

The center of gravity of a body or a system of particles rigidly connected together, is that point through which the line of action of the weight of the body always passes in whatever position the body is placed and this point is called centroid. A body can have one and only one center of gravity.

If $w_1, w_2, \ldots\ldots\ldots .., w_n$ are the weights of the particles placed at the points $A_1(x_1, y_1), A_2(x_2, y_2), \ldots\ldots\ldots .., A_n(x_n, y_{n)}$ respectively, then the center of gravity $G(\bar{x}, \bar{y})$ is given by $\bar{x} = \frac{\sum w_1 x_1}{\sum w_1}, \bar{y} = \frac{\sum w_1 y_1}{\sum w_1}$.

(1) **Centre of gravity of a number of bodies of different shape:**

(i) **C.G. of a uniform rod:** The C.G. of a uniform rod lies at its mid-point.

(ii) **C.G. of a uniform parallelogram:** The C.G. of a uniform parallelogram is the point of intersection of the diagonals.

(iii) **C.G. of a uniform triangular lamina:** The C.G. of a triangle lies on a median at a distance from the base equal to one third of the medians.

(2) **Some Important points to remember:**

(i) The C.G. of a uniform tetrahedron lies on the line joining a vertex to the C.G. of the opposite face, dividing this line in the ratio 3 : 1.

(ii) The C.G. of a right circular solid cone lies at a distance h/4 from the base on the axis and divides it in the ratio 3 : 1.

(iii) The C.G. of the curved surface of a right circular hollow cone lies at a distance h/3 from the base on the axis and divides it in the ratio 2 : 1

(iv) The C.G. of a hemispherical shell at a distance a/2 from the center on the symmetrical radius.

(v) The C.G. of a solid hemisphere lies on the central radius at a distance 3a/8 from the center where a is the radius.

(vi) The C.G. of a circular arc subtending an angle 2α at the center is at a distance $\frac{a \sin \alpha}{\alpha}$ from the center on the symmetrical radius, a being the radius, and α in radians.

(vii) The C.G. of a sector of a circle subtending an angle 2α at the center is at a distance $\frac{2a}{3} \frac{\sin \alpha}{\alpha}$ from the center on the symmetrical radius, a being the radius and α in radians.

(viii) The C.G. of the semicircular arc lies on the central radius at a distance of $\frac{2a}{\pi}$ from the boundary diameter, where a is the radius of the arc.

Important Tips

- ☞ Let there be a body of weight w and x be its C.G. If a portion of weight w_1 is removed from it and x_1 be the C.G. of the removed portion. Then, the C.G. of the remaining portion is given by $x_2 = \frac{wx - w_1 x_1}{w - w_1}$
- ☞ Let x be the C.G. of a body of weight w. If x_1, x_2, x_3 are the C.G. of portions of weights w_1, w_2, w_3 respectively, which are removed from the body, then the C.G. of the remaining body is given by $x_4 = \frac{wx - w_1 x_1 - w_2 x_2 - w_3 x_3}{w - w_1 - w_2 - w_3}$