Parallelogram law of Forces.

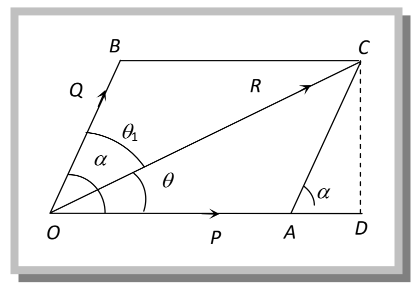
If two forces, acting at a point, be represented in magnitude and direction by the two sides of a parallelogram drawn from one of its angular points, their resultant is represented both in magnitude and direction of the parallelogram drawn through that point.

If OA and OB represent the forces P and Q acting at a point O and inclined to each other at an angle α. If R is the resultant of these forces represented by the diagonal OC of the parallelogram OACB and R makes an angle θ with P i.e. $\angle COA = \theta$, then $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$ and

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

The angle θ_1 which the resultant R makes with the direction of the force Q is given by

$$\theta_1 = \tan^{-1}\left(\frac{P\sin\alpha}{Q + P\cos\alpha}\right)$$



Case (i): If P = Q

$\therefore R = 2P\cos(\alpha/2)$ and $\tan\theta = \tan(\alpha/2)$ or $\theta = \alpha/2$

Case (ii): If $\alpha = 90°$, i.e. forces are perpendicular

$\therefore R = \sqrt{P^2 + Q^2}$ and $\tan\theta = \dfrac{Q}{P}$

Case (iii): If $\alpha = 0°$, i.e. forces act in the same direction

$\therefore R_{max} = P + Q$

Case (iv): If $\alpha = 180°$, i.e. forces act in opposite direction

$\therefore R_{min} = P - Q$

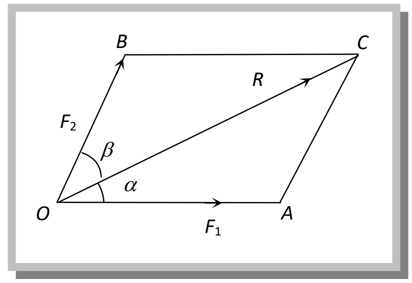
Note: The resultant of two forces is closer to the larger force.

The resultant of two equal forces of magnitude P acting at an angle α is 2P cos $\dfrac{\alpha}{2}$ and it bisects the angle between the forces.

If the resultant R of two forces P and Q acting at an angle α makes an angle θ with the direction of P, then

$\sin\theta = \dfrac{Q\sin\alpha}{R}$ and $\cos\theta = \dfrac{P + Q\cos\alpha}{R}$

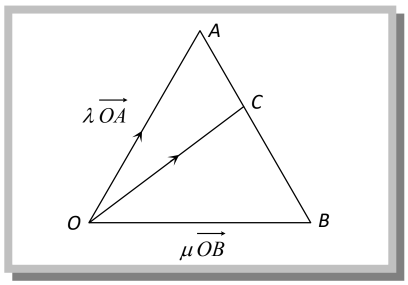
If the resultant R of the forces P and Q acting at an angle α makes an angle θ with the direction of the force Q, then $\sin\theta = \frac{P\sin\alpha}{R}$ and $\cos\theta = \frac{Q + P\sin\alpha}{R}$



Component of a force in two directions: The component of a force R in two directions making angles α and β with the line of action of R on and opposite sides of it are

$$F_1 = \frac{OC\,.\sin\beta}{\sin(\alpha+\beta)} = \frac{R\sin\beta}{\sin(\alpha+\beta)} \text{ and } F_2 = \frac{OC.\sin\alpha}{\sin(\alpha+\beta)} = \frac{R.\sin\alpha}{\sin(\alpha+\beta)}$$

λ-μ theorem : The resultant of two forces acting at a point O in directions OA and OB represented in magnitudes by λ.OA and μ.OB respectively is represented by $(\lambda+\mu)OC$, where C is a point in AB such that $\lambda.CA = \mu.CB$



Important Tips

☞ The forces P, Q, R act along the sides BC, CA, AB of ΔABC.

Their resultant passes through.

(a) Incentre, if $P + Q + R = 0$ (b) Circumcentre, if

$P\cos A + Q\cos B + R\cos C = 0$

(c) Orthocentre, if $P\sec A + Q\sec B + R\sec C = 0$ (d) Centroid, if

$P\,\text{cosec}\,A + Q\,\text{cosec}\,B + R\,\text{cosec}\,C = 0$

or $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$