Numerical Integration.

It is the process of computing the value of a definite integral when we are given a set of numerical values of the integrand f(x) corresponding to some values of the independent variable x.

If $I = \int_a^b y.dx$. Then I represents the area of the region R under the curve $y = f(x)$ between the ordinates $x = a, x = b$ and the x-axis.

(1) **Trapezoidal rule**

Let $y = f(x)$ be a function defined on [a, b] which is divided into n equal sub-intervals each of width h so that $b - a = nh$.

Let the values of $f(x)$ for $(n+1)$ equidistant arguments

$x_0 = a,\ x_1 = x_0 + h,\ \ x_2 = x_0 + 2h, \ldots\ldots\ldots,\ x_n = x_0 + nh = b$ be $y_0, y_1, y_2, \ldots\ldots\ldots\ y_n$ respectively.

Then $\int_a^b f(x)dx = \int_{x_0}^{x_0+nh} y\,dx = h\left[\frac{1}{2}(y_0 + y_n) + (y_1 + y_2 + \ldots\ldots + y_{n-1})\right]$

$= \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots\ldots + y_{n-1})]$

This rule is known as **Trapezoidal rule.**

The geometrical significance of this rule is that the curve $y = f(x)$ is replaced by n straight lines joining the points (x_0, y_0) and (x_1, y_1); (x_1, y_1) and (x_2, y_2); (x_{n-1}, y_{n-1}) and (x_n, y_n). The area bounded by the curve $y = f(x)$. The ordinate $x = x_0$ and $x = x_n$ and the x-axis, is then approximately equivalent to the sum of the areas of the n trapeziums obtained.

(2) **Simpson's one third rule:** Let $y = f(x)$ be a function defined on [a, b] which is divided into n (an even number) equal parts each of width h so that $b - a = nh$.

Suppose the function $y = f(x)$ attains values $y_0, y_1, y_2, \ldots\ldots\ldots .\ y_n$ at $n+1$ equidistant points $x_0 = a,\ \ x_1 = x_0 + h$, $x_2 = x_0 + 2h, \ldots\ldots\ldots\ \ldots,\ \ x_n = x_0 + nh = b$ respectively. Then

$$\int_a^b f(x)dx = \int_{x_0}^{x_0+nh} ydx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \ldots. + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})]$$

= (one-third of the distance between two consecutive ordinates)

$\times$ [(sum of the extreme ordinates)+4(sum of odd ordinates)+2(sum of even ordinates)]

This formula is known as Simpson's one-third rule. Its geometric significance is that we replace the graph of the given function by $\frac{n}{2}$ arcs of second degree polynomials, or parabolas with vertical axes. It is to note here that the interval [a, b] is divided into an even number of subinterval of equal width.

Simpson's rule yield more accurate results than the trapezoidal rule. Small size of interval gives more accuracy.