Number of Combinations without Repetition.

The number of combinations (selections or groups) that can be formed from n different objects taken $r (0 \le r \le n)$ at a time is ${}^nC_r = \frac{n!}{r!(n-r)!}$

Let the total number of selections (or groups) = x. Each group contains r objects, which can be arranged in r !Ways. Hence the number of arrangements of r objects = $x \times (r!)$. But the number of arrangements = ${}^nP_r$.

$$\Rightarrow x \times (r!) = {}^nP_r \Rightarrow x = \frac{{}^nP_r}{r!} \Rightarrow x = \frac{n!}{r!(n-r)!} = {}^nC_r .$$

Important Tips

- ☞ ${}^nC_r$ is a natural number.
- ☞ ${}^nC_0 = {}^nC_n = 1, {}^nC_1 = n$
- ☞ ${}^nC_r = {}^nC_{n-r}$
- ☞ ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ☞ ${}^nC_x = {}^nC_y \Leftrightarrow x = y$ or $x + y = n$
- ☞ $n . {}^{n-1}C_{r-1} = (n-r+1)^n C_{r-1}$
- ☞ If n is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.
- ☞ If n is odd then the greatest value of ${}^nC_r$ is $\frac{{}^nC_{n+1}}{2}$ or $\frac{{}^nC_{n-1}}{2}$.
- ☞ ${}^nC_r = \frac{n}{r} . {}^{n-1}C_{r-1}$
- ☞ $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$
- ☞ ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$
- ☞ ${}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}$
- ☞ ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n = 2^{2n}$
- ☞ ${}^nC_n + {}^{n+1}C_n + {}^{n+2}C_n + {}^{n+3}C_n + ... + {}^{2n-1}C_n = {}^{2n}C_{n+1}$

Note: Number of combinations of n dissimilar things taken all at a time ${}^nC_n = \frac{n!}{n!(n-n)!} = \frac{1}{0!} = 1$, $(\because 0! = 1)$.