## Division into Groups.

**Case I:**(1) The number of ways in which n different things can be arranged into r different groups is $^{n+r-1}P_n$ or n ! $^{n-1}C_{r-1}$ According as blank group are or are not admissible.

(2) The number of ways in which n different things can be distributed into r different group is $r^n - {}^rC_1(r-1)^n + {}^rC_2(r-2)^n - ......... + (-1)^{n-1}\ {}^nC_{r-1}$ or Coefficient of $x^n$ is n ! $(e^x - 1)^r$
Here blank groups are not allowed.

(3) Number of ways in which m × n different objects can be distributed equally among n persons (or numbered groups) = (number of ways of dividing into groups) × (number of groups) ! = $\frac{(mn)!n!}{(m!)^n n!} = \frac{(mn)!}{(m!)^n}$.

**Case II:**(1) The number of ways in which $(m+n)$ different things can be divided into two groups which contain m and n things respectively is, $^{m+n}C_m . ^nC_n = \frac{(m+n)!}{m!n!}, m \neq n$.

**Corollary:** If $m = n$, then the groups are equal size. Division of these groups can be given by two types.

**Type I: If order of group is not important:**The number of ways in which 2n different things can be divided equally into two groups is $\frac{(2n)!}{2!(n!)^2}$

**Type II:If order of group is important:** The number of ways in which 2n different things can be divided equally into two distinct groups is $\frac{(2n)!}{2!(n!)^2} \times 2! = \frac{2n!}{(n!)^2}$

(2) The number of ways in which (m + n + p) different things can be divided into three groups which contain m, n and p things respectively is $^{m+n+p}C_m . ^{n+p}C_n . ^pC_p = \frac{(m+n+p)!}{m!n!p!}, m \neq n \neq p$

**Corollary:** If $m = n = p$, then the groups are equal size. Division of these groups can be given by two types.

**Type I: If order of group is not important:** The number of ways in which 3p different things can be divided equally into three groups is $\frac{(3p)!}{3!(p!)^3}$

**Type II:If order of group is important:** The number of ways in which 3p different things can be divided equally into three distinct groups is $\frac{(3p)!}{3!(p!)^3} t 3! = \frac{(3p)!}{(p!)^3}$

Note: If order of group is not important: The number of ways in which mn different things can be divided equally into m groups is $\frac{mn!}{(n!)^m m!}$

If order of group is important: The number of ways in which mn different things can be divided equally into m distinct groups is $\frac{(mn)!}{(n!)^m m!} \times m! = \frac{(mn)!}{(n!)^m}$.