Number of Divisors.

Let $N = p_1^{\alpha_1}.p_2^{\alpha_2}.p_3^{\alpha_3}......p_k^{\alpha_k}$, where $p_1, p_2, p_3,......p_k$ are different primes and $\alpha_1, \alpha_2, \alpha_3,......, \alpha_k$ are natural numbers then:

(1) The total number of divisors of N including 1 and N is = $(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)....(\alpha_k + 1)$

(2) The total number of divisors of N excluding 1 and N is =
$(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1).....(\alpha_k + 1) - 2$

(3) The total number of divisors of N excluding 1 or N is = $(\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1).....(\alpha_k + 1) - 1$

(4) The sum of these divisors is
$= (p_1^0 + p_2^1 + p_3^2 + ... + p_1^{\alpha_1})(p_2^0 + p_2^1 + p_2^2 + ... + p_2^{\alpha_2}).....(p_k^0 + p_k^1 + p_k^2 + ... + p_k^{\alpha_k})$

(5) The number of ways in which N can be resolved as a product of two factors is

$$\begin{cases} \frac{1}{2}(\alpha_1 + 1)(\alpha_2 + 1)....(\alpha_k + 1), \text{If } N \text{ is not a perfect square} \\ \frac{1}{2}[(\alpha_1 + 1)(\alpha_2 + 1).....(\alpha_k + 1) + 1], \text{If } N \text{ is a perfect square} \end{cases}$$

(6) The number of ways in which a composite number N can be resolved into two factors which are relatively prime (or co-prime) to each other is equal to 2^{n-1} where n is the number of different factors in N.

Important Tips

- All the numbers whose last digit is an even number 0, 2, 4, 6 or 8 are divisible by 2.
- All the numbers sum of whose digits are divisible by 3, is divisible by 3 e.g. 534. Sum of the digits is 12, which are divisible by 3, and hence 534 is also divisible by 3.
- All those numbers whose last two-digit number is divisible by 4 are divisible by 4 e.g. 7312, 8936, are such that 12, 36 are divisible by 4 and hence the given numbers are also divisible by 4.
- All those numbers, which have either 0 or 5 as the last digit, are divisible by 5.
- All those numbers, which are divisible by 2 and 3 simultaneously, are divisible by 6. e.g., 108, 756 etc.
- All those numbers whose last three-digit number is divisible by 8 are divisible by 8.
- All those numbers sum of whose digit is divisible by 9 are divisible by 9.

☞ All those numbers whose last two digits are divisible by 25 are divisible by 25 e.g., 73125, 2400 etc.