Exponent of Prime p in n !

Let p be a prime number and n be a positive integer. Then the last integer amongst 1, 2, 3, ...(n – 1), n which is divisible by p is $\left[\frac{n}{p}\right]p$, where $\left[\frac{n}{p}\right]$ denote the greatest integer less than or equal to $\frac{n}{p}$.

For example: $\left[\frac{10}{3}\right] = 3$, $\left[\frac{12}{5}\right] = 2$, $\left[\frac{15}{3}\right] = 5$ etc.

Let $E_p(n)$ denotes the exponent of the prime p in the positive integer n. Then,

$$E_p(n!) = E_p(1.2.3.........(n-1)n) = E_p\left(p.2p.3p.......\left[\frac{n}{p}\right]p\right) = \left[\frac{n}{p}\right] + E_p\left(1.2.3......\left[\frac{n}{p}\right]\right)$$

[∵ Remaining integers between 1 and n are not divisible by p]

Now the last integer amongst 1, 2, 3,..... $\left[\frac{n}{p}\right]$ Which is divisible by p is

$\left[\frac{n/p}{p}\right] = \left[\frac{n}{p^2}\right] = \left[\frac{n}{p}\right] + E_p\left(p, 2p, 3p....\left[\frac{n}{p^2}\right]p\right)$ Because the remaining natural numbers from 1 to $\left[\frac{n}{p}\right]$ are not divisible by p = $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + E_p\left(1.2.3......\left[\frac{n}{p^2}\right]\right)$

Similarly we get $E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + ...\left[\frac{n}{p^S}\right]$

where S is the largest natural number. Such that $p^S \le n < p^{S+1}$.