Conditional Permutations.

(1) Number of permutations of n dissimilar things taken r at a time when p particular things always occur = ${}^{n-p}C_{r-p}\, r!$

(2) Number of permutations of n dissimilar things taken r at a time when p particular things never occur = ${}^{n-p}C_r\, r!$

(3) The total number of permutations of n different things taken not more than r at a time, when each thing may be repeated any number of times, is $\frac{n(n^r-1)}{n-1}$.

(4) Number of permutations of n different things, taken all at a time, when m specified things always come together is $m! \times (n-m+1)!$

(5) Number of permutations of n different things, taken all at a time, when m specified things never come together is $n! - m! \times (n-m+1)!$

(6) Let there be n objects, of which m objects are alike of one kind, and the remaining $(n-m)$ objects are alike of another kind. Then, the total number of mutually distinguishable permutations that can be formed from these objects is $\frac{n!}{(m!) \times (n-m)!}$.

Note: The above theorem can be extended further i.e., if there are n objects, of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of 3rd kind;……: p_r are alike of rth kind such that $p_1 + p_2 + \ldots\ldots + p_r = n$; then the number of permutations of these n objects is

$$\frac{n!}{(p_1!) \times (p_2!) \times \ldots\ldots \times (p_r!)}.$$

Important Tips

☞ **Gap method:** Suppose 5 males A, B, C, D, E are arranged in a row as × A × B × C × D × E ×. There will be six gaps between these five. Four in between and two at either end. Now if three females P, Q, R are to be arranged so that no two are together we shall use gap method i.e., arrange them in between these 6 gaps. Hence the answer will be ${}^6P_3$.

☞ **Together:** Suppose we have to arrange 5 persons in a row which can be done in 5 ! = 120 ways. But if two particular persons are to be together always, then we tie these two particular persons with a string. Thus we have 5 – 2 + 1 (1 corresponding to these two together) = 3 +1 = 4 units, which can be arranged in 4! ways. Now we loosen the string and these two particular can be arranged in 2 !ways. Thus total arrangements = 24 × 2 = 48.

Never together = Total – Together = 120 – 48 = 72.

Ways. Hence the required number of ways = 6 × 3 = 18.