Circular Permutations.

So far we have been considering the arrangements of objects in a line. Such permutations are known as linear permutations.

Instead of arranging the objects in a line, if we arrange them in the form of a circle, we call them, circular permutations.

In circular permutations, what really matters is the position of an object relative to the others.

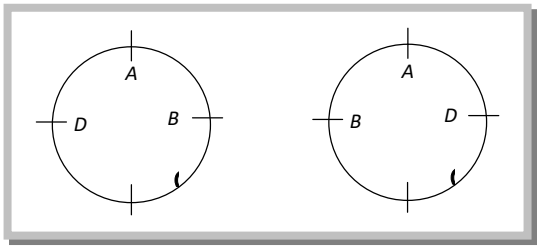
Thus, in circular permutations, we fix the position of the one of the objects and then arrange the other objects in all possible ways.

There are two types of circular permutations:

(i) The circular permutations in which clockwise and the anticlockwise arrangements give rise to different permutations, e.g. seating arrangements of persons round a table.

(ii) The circular permutations in which clockwise and the anticlockwise arrangements give rise to same permutations, e.g. arranging some beads to form a necklace.

Look at the circular permutations, given below:



Suppose A, B, C, D are the four beads forming a necklace. They have been arranged in clockwise and anticlockwise directions in the first and second arrangements respectively.

Now, if the necklace in the first arrangement be given a turn, from clockwise to anticlockwise, we obtain the second arrangement. Thus, there is no difference between the above two arrangements.

(1) **Difference between clockwise and anticlockwise arrangement :** If anticlockwise and clockwise order of arrangement are not distinct e.g., arrangement of beads in a necklace, arrangement of flowers in garland etc. then the number of circular permutations of n distinct items is $\frac{(n-1)!}{2}$

(2) **Theorem on circular permutations**

Theorem 1: The number of circular permutations of n different objects is $(n-1)!$

Theorem 2: The number of ways in which n persons can be seated round a table is $(n-1)!$

Theorem 3: The number of ways in which n different beads can be arranged to form a necklace, is $\frac{1}{2}(n-1)!$.

Note: When the positions are numbered, circular arrangement is treated as a linear arrangement. In a linear arrangement, it does not make difference whether the positions are numbered or not.