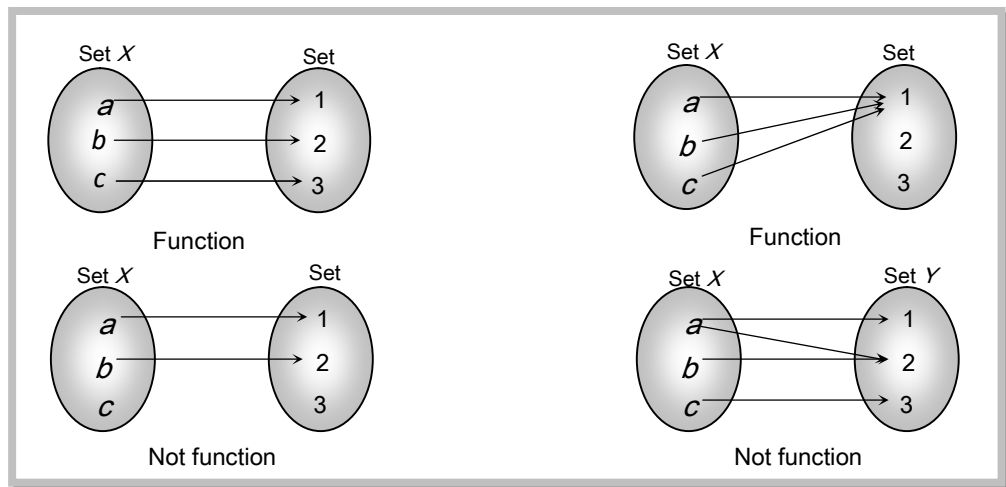


## Definition of Function.

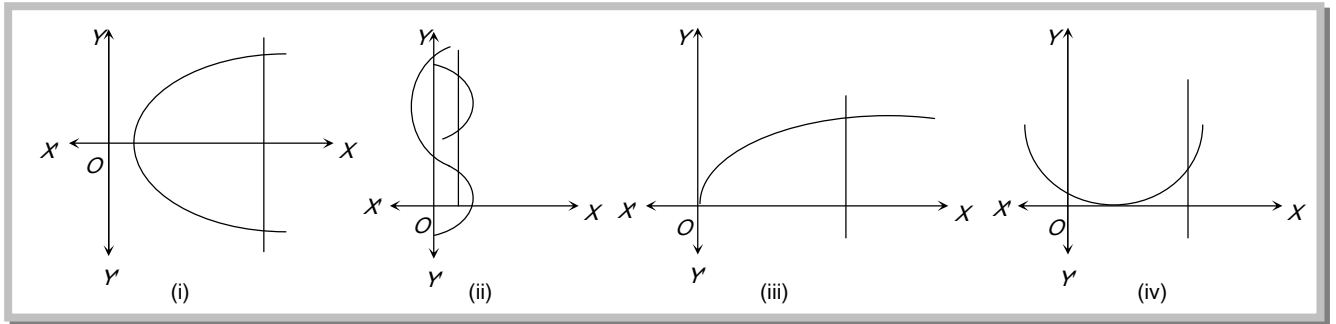
(1) Function can be easily defined with the help of the concept of mapping. Let  $X$  and  $Y$  be any two non-empty sets. "A function from  $X$  to  $Y$  is a rule or correspondence that assigns to each element of set  $X$ , one and only one element of set  $Y$ ". Let the correspondence be ' $f$ ' then mathematically we write  $f: X \rightarrow Y$  where  $y = f(x), x \in X$  and  $y \in Y$ . We say that ' $y$ ' is the image of ' $x$ ' under  $f$  (or  $x$  is the pre image of  $y$ ).

Two things should always be kept in mind:

- (i) A mapping  $f: X \rightarrow Y$  is said to be a function if each element in the set  $X$  has its image in set  $Y$ . It is also possible that there are few elements in set  $Y$  which are not the images of any element in set  $X$ .
- (ii) Every element in set  $X$  should have one and only one image. That means it is impossible to have more than one image for a specific element in set  $X$ . Functions cannot be multi-valued (A mapping that is multi-valued is called a relation from  $X$  and  $Y$ ) e.g.



(2) **Testing for a function by vertical line test:** A relation  $f: A \rightarrow B$  is a function or not it can be checked by a graph of the relation. If it is possible to draw a vertical line which cuts the given curve at more than one point then the given relation is not a function and when this vertical line means line parallel to  $Y$ -axis cuts the curve at only one point then it is a function. Figure (iii) and (iv) represents a function.



(3) **Number of functions:** Let  $X$  and  $Y$  be two finite sets having  $m$  and  $n$  elements respectively. Then each element of set  $X$  can be associated to any one of  $n$  elements of set  $Y$ . So, total number of functions from set  $X$  to set  $Y$  is  $n^m$ .

(4) Value of the function: If  $y = f(x)$  is a function then to find its values at some value of  $x$ , say  $x = a$ , we directly substitute  $x = a$  in its given rule  $f(x)$  and it is denoted by  $f(a)$ .

*e.g.* If  $f(x) = x^2 + 1$ , then  $f(1) = 1^2 + 1 = 2$ ,  $f(2) = 2^2 + 1 = 5$ ,  $f(0) = 0^2 + 1 = 1$  etc.