# Algebra of Functions.

Let $f(x)$ and $g(x)$ be two real and single-valued functions, with domains $X_f, X_g$ and ranges $Y_f$ and $Y_g$ respectively. Let $X = X_f \cap X_g \neq \phi$. Then, the following operations are defined.

(1) **Scalar multiplication of a function:** $(c\,f)(x) = c\,f(x)$, where $c$ is a scalar. The new function $c\,f(x)$ has the domain $X_f$.

(2) **Addition/subtraction of functions:** $(f \pm g)(x) = f(x) \pm g(x)$. The new function has the domain $X$.

(3) **Multiplication of functions:** $(fg)(x) = (g\,f)(x) = f(x)g(x)$. The product function has the domain $X$.

(4) **Division of functions:**

(i) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$. The new function has the domain $X$, except for the values of $x$ for which $g(x) = 0$.

(ii) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$. The new function has the domain $X$, except for the values of $x$ for which $f(x) = 0$.

(5) **Equal functions:** Two function $f$ and $g$ are said to be equal functions, if and only if

(i) Domain of $f$ = domain of $g$

(ii) Co-domain of $f$ = co-domain of $g$

(iii) $f(x) = g(x) \forall x \in$ their common domain

(6) **Real valued function:** If $R$, be the set of real numbers and $A$, $B$ are subsets of $R$, then the function $f : A \rightarrow B$ is called a real function or real –valued function.