Periodic Function.

A function is said to be periodic function if its each value is repeated after a definite interval. So a function *f*(*x*) will be periodic if a positive real number *T* exist such that, $f(x+T)=f(x),\ \forall x \in$ domain. Here the least positive value of *T* is called the period of the function. Clearly $f(x)=f(x+T)=f(x+2T)=f(x+3T)=.....$. *E.g.* $\sin x, \cos x, \tan x$ are periodic functions with period $2\pi, 2\pi$ and π respectively.

Some standard results on periodic functions

Functions	**Periods**
(1) $\sin^n x,\ \cos^n x$, $\sec^n x,\ cosec^n x$	π; if *n* is even 2π; if *n* is odd or fraction
(2) $\tan^n x,\ \cot^n x$	π; *n* is even or odd.
(3) $\lvert \sin x \rvert, \lvert \cos x \rvert, \lvert \tan x \rvert,$ $\lvert \cot x \rvert, \lvert \sec x \rvert, \lvert cosec\ x \rvert$	π
(4) $x-[x]$	1
(5) Algebraic functions e.g., $\sqrt{x}, x^2, x^3+5,....$etc	Period does not exist

Important Tips

- *If f(x) is periodic with period T, then c.f(x) is periodic with period T, f(x + c) is periodic with period T and f(x) ± c is periodic with period T. where c is any constant.*
- *If a function f(x) has a period T, then the function f (ax+b) will have a period* $\frac{T}{|a|}$.
- *If f(x) is periodic with period T then* $\frac{1}{f(x)}$ *is also periodic with same period T.*
- *If f(x) is periodic with period T,* $\sqrt{f(x)}$ *is also periodic with same period T.*
- *If f(x) is periodic with period T, then a f(x) + b, where* $a, b \in R\ (a \neq 0)$ *is also a periodic function with periodT.*

☞ *If $f_1(x)$, $f_2(x)$, $f_3(x)$ are periodic functions with periods T_1, T_2, T_3 respectively then; we have* $h(x) = af_1(x) \pm bf_2(x) \pm cf_3(x)$, *has period as,*

$$= \begin{cases} \text{L.C.M.of } \{T_1, T_2, T_3\}; & \text{if } h(x) \text{is not an even function} \\ \frac{1}{2}\text{L.C.M. of } \{T_1, T_2, T_3\}; & \text{if } h(x) \text{ is an even function} \end{cases}$$