Composite Function.

If $f : A \to B$ and $g : B \to C$ are two function then the composite function of *f* and *g*, $gof\ A \to C$ will be defined as $gof(x) = g[f(x)], \forall x \in A$

(1) **Properties of composition of function:**

(i) *f* is even, *g* is even $\Rightarrow$ fog even function.

(ii) *f* is odd, *g* is odd $\Rightarrow$ fog is odd function.

(iii) *f* is even, *g* is odd $\Rightarrow$ fog is even function.

(iv) *f* is odd, *g* is even $\Rightarrow$ fog is even function.

(v) Composite of functions is not commutative *i.e.* $fog \neq gof$

(vi) Composite of functions is associative *i.e.* $(fog)oh = fo(goh)$

(vii) If $f : A \to B$ is bijection and $g : B \to A$ is inverse of *f*. Then $fog = I_B$ and $gof = I_A$.
Where, I_A and I_B are identity functions on the sets *A* and *B* respectively.

(viii) If $f : A \to B$ and $g : B \to C$ are two bijections, then $gof : A \to C$ is bijection and $(gof)^{-1} = (f^{-1} o g^{-1})$.

(ix) $fog \neq gof$ but if , $fog = gof$ then either $f^{-1} = g$ or $g^{-1} = f$ also, $(fog)(x) = (gof)(x) = (x)$.

Important Tips

- *gof(x) is simply the g-image of f(x), where f(x) is f-image of elements x∈ A.*
- *Function gof will exist only when range of f is the subset of domain of g.*
- *fog does not exist if range of g is not a subset of domain of f.*
- *fog and gof may not be always defined.*
- *If both f and g are one-one, then fog and gof are also one-one.*
- *If both f and g are onto, then gof is onto.*