Limit of a Function.

Let $y = f(x)$ be a function of x. If at $x = a, f(x)$ takes indeterminate form, then we consider the values of the function which are very near to 'a'. If these values tend to a definite unique number as x tends to 'a', then the unique number so obtained is called the limit of $f(x)$ at $x = a$ and we write it as $\lim_{x \to a} f(x)$.

(1) **Meaning of 'x $\to$ a':** Let x be a variable and a be the constant. If x assumes values nearer and nearer to 'a' then we say 'x tends to a' and we write '$x \to a$'. It should be noted that as $x \to a$, we have $x \neq a$. By 'x tends to a' we mean that

(i) $x \neq a$

(ii)x assumes values nearer and nearer to 'a' and

(iii) We are not specifying any manner in which x should approach to 'a'. x may approach to a from left or right as shown i

(2) **Left hand and right hand limit:**Consider the values of the functions at the points which are very near to a on the left of a. If these values tend to a definite unique number as x tends to a, then the unique number so obtained is called left-hand limit of $f(x)$ at x = a and symbolically we write it as $f(a-0) = \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h)$

Similarly we can define right-hand limit of $f(x)$ at $x = a$ which is expressed as

$f(a+0) = \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h)$.

(3) **Method for finding L.H.L. and R.H.L.**

(i) For finding right hand limit (R.H.L.) of the function, we write x + h in place of x, while for left hand limit (L.H.L.) we write x – h in place of x.

(ii) Then we replace x by 'a' in the function so obtained.

(iii) Lastly we find limit $h \to 0$.

(4) **Existence of limit:** $\lim_{x \to a} f(x)$ exists when,

(i) $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist i.e. L.H.L. and R.H.L. both exists.

(ii) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ i.e. L.H.L. = R.H.L.

Note: If a function $f(x)$ takes the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ at $x = a$, then we say that $f(x)$ is indeterminate or meaningless at $x = a$. Other indeterminate forms are $\infty - \infty, \infty \times \infty, 0 \times \infty, \ 1^{\infty}, 0^{0}, \infty^{0}$

In short, we write L.H.L. for left hand limit and R.H.L. for right hand limit.

It is not necessary that if the value of a function at some point exists then its limit at that point must exist.

(5) **Sandwich theorem:** If $f(x)$, $g(x)$ and $h(x)$ are any three functions such that, $f(x) \le g(x) \le h(x) \ \forall x \in$ neighborhood of $x = a$ and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = l\,(\text{say})$, then $\lim_{x \to a} g(x) = l.$

This theorem is normally applied when the $\lim_{x \to a} g(x)$ can't be obtained by using conventional methods as function $f(x)$ and $h(x)$ can be easily found.