Fundamental Theorems on Limits.

The following theorems are very useful for evaluation of limits if $\lim\limits_{x\to 0} f(x) = l$ and $\lim\limits_{x\to 0} g(x) = m$ (l and m are real numbers) then

(1) $\lim\limits_{x\to a}(f(x) + g(x)) = l + m$ (Sum rule)

(2) $\lim\limits_{x\to a}(f(x) - g(x)) = l - m$ (Difference rule)

(3) $\lim\limits_{x\to a}(f(x).g(x)) = l.m$ (Product rule)

(4) $\lim\limits_{x\to a} k\ f(x) = k.l$ (Constant multiple rule)

(5) $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \frac{l}{m}, m \neq 0$ (Quotient rule)

(6) If $\lim\limits_{x\to a} f(x) = +\infty$ or $-\infty$, then $\lim\limits_{x\to a} \frac{1}{f(x)} = 0$

(7) $\lim\limits_{x\to a} \log\{f(x)\} = \log\{\lim\limits_{x\to a} f(x)\}$

(8) If $f(x) \leq g(x)$ for all x, then $\lim\limits_{x\to a} f(x) \leq \lim\limits_{x\to a} g(x)$

(9) $\lim\limits_{x\to a}[f(x)]^{g(x)} = \{\lim\limits_{x\to a} f(x)\}^{\lim\limits_{x\to a} g(x)}$

(10) If p and q are integers, then $\lim\limits_{x\to a}(f(x))^{p/q} = l^{p/q}$, provided $(l)^{p/q}$ is a real number.

(11) If $\lim\limits_{x\to a} f(g(x)) = f(\lim\limits_{x\to a} g(x)) = f(m)$ provided ‘f’ is continuous at $g(x) = m.\ e.g.$ $\lim\limits_{x\to a} \ln[f(x)] = \ln(l),$ only if $l > 0.$