Methods of Evaluation of Limits.

We shall divide the problems of evaluation of limits in five categories.

(1) **Algebraic limits:** Let $f(x)$ be an algebraic function and 'a' be a real number. Then $\lim_{x \to a} f(x)$ is known as an algebraic limit.

(i) **Direct substitution method:** If by direct substitution of the point in the given expression we get a finite number, then the number obtained is the limit of the given expression.

(ii) **Factorization method:** In this method, numerator and denominator are factorized. The common factors are cancelled and the rest outputs the results.

(iii) **Rationalization method:** Rationalization is followed when we have fractional powers (like $\frac{1}{2}, \frac{1}{3}$ etc.) on expressions in numerator or denominator or in both. After rationalization the terms are factorized which on cancellation gives the result.

(iv) **Based on the form when x $\to \infty$:** In this case expression should be expressed as a function 1/x and then after removing indeterminate form, (if it is there) replace $\frac{1}{x}$ by 0.

Step I: Write down the expression in the form of rational function, i.e., $\frac{f(x)}{g(x)}$, if it is not so.

Step II: If k is the highest power of x in numerator and denominator both, then divide each term of numerator and denominator by x^k.

Step III: Use the result $\lim_{x \to \infty} \frac{1}{x^n} = 0$, where n > 0.

Note: **An important result:** If m, n are positive integers and $a_0, b_0 \neq 0$ are non-zero real numbers, then

$$\lim_{x \to \infty} \frac{a_0 x^m + a_1 x^{m-1} + ... + a_{m-1} x + a_m}{b_0 x^n + b_1 x^{n-1} + ... + b_{n-1} x + b_n} = \begin{cases} \frac{a_0}{b_0}, \text{ if } m = n \\ 0, \text{ if } m < n \\ \infty, \text{if } m > n \end{cases}$$