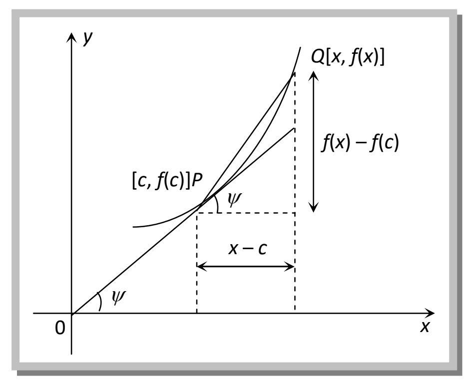
Derivative at a Point.

The derivative of a function at a point $x = a$ is defined by $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ (provided the limit exists and is finite)

The above definition of derivative is also called derivative by first principle.

(1) **Geometrical meaning of derivatives at a point:** Consider the curve $y = f(x)$. Let $f(x)$ be differentiable at $x = c$. Let $P(c, f(c))$ be a point on the curve and $Q(x, f(x))$ be a neighboring point on the curve. Then,



Slope of the chord $PQ = \frac{f(x) - f(c)}{x - c}$. Taking limit as $Q \to P$, *i.e.* $x \to c$,

We get $\lim_{Q \to P}$ (slope of the chord PQ) $= \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ ......(i)

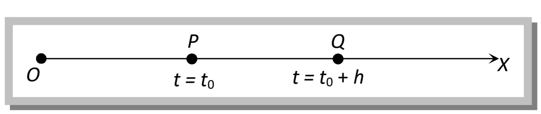
As $Q \to P$, chord *PQ* becomes tangent at *P*.

Therefore from (i), we have

Slope of the tangent at *P*= $\lim_{x \to c} \frac{f(x) - f(c)}{x - c} = \left(\frac{df(x)}{dx}\right)_{x=c}$.

Note: Thus, the derivatives of a function at a point $x = c$ is the slope of the tangent to curve, $y = f(x)$ at point $(c, f(c))$.

(2) **Physical interpretation at a point:** Let a particle moves in a straight line *OX* starting from *O* towards *X*. Clearly, the position of the particle at any instant would depend upon the time elapsed. In other words, the distance of the particle from *O* will be some function *f* of time *t*.



Let at any time $t = t_0$, the particle be at *P* and after a further time *h*, it is at *Q* so that *OP* = $f(t_0)$ and $OQ = f(t_0 + h)$. Hence, the average speed of the particle during the journey from *P* to *Q* is $\frac{PQ}{h}$, *i.e.,* $\frac{f(t_0 + h) - f(t_0)}{h} = f(t_0, h)$. Taking the limit of $f(t_0, h)$ as $h \to 0$, we get its instantaneous speed to be $\lim_{h\to 0}\frac{f(t_0 + h) - f(t_0)}{h}$, which is simply $f'(t_0)$. Thus, if $f(t)$ gives the distance of a moving particle at time *t*, then the derivative of *f* at $t = t_0$ represents the instantaneous speed of the particle at the point *P*, *i.e.,* at time $t = t_0$.

Important Tips

- *$\frac{dy}{dx}$ is $\frac{d}{dx}(y)$ in which $\frac{d}{dx}$ is simply a symbol of operation and not 'd' divided by dx.*
- *If $f'(x_0) = \infty$, the function is said to have an infinite derivative at the point x_0. In this case the line tangent to the curve of $y = f(x)$ at the point x_0 is perpendicular to the x-axis*
