Some Standard Differentiation.

(1) **Differentiation of algebraic functions**

(i) $\frac{d}{dx}x^n = nx^{n-1}, x \in R, n \in R, x > 0$ (ii) $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$ (iii) $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$

(2) **Differentiation of trigonometric functions:**The following formulae can be applied directly while differentiating trigonometric functions

(i) $\frac{d}{dx}\sin x = \cos x$

(ii) $\frac{d}{dx}\cos x = -\sin x$

(iii) $\frac{d}{dx}\tan x = \sec^2 x$

(iv) $\frac{d}{dx}\sec x = \sec x \tan x$

(v) $\frac{d}{dx}\text{cosec } x = -\text{cosec } x \cot x$

(vi) $\frac{d}{dx}\cot x = -\text{cosec}^2 x$

(3) **Differentiation of logarithmic and exponential functions:**The following formulae can be applied directly when differentiating logarithmic and exponential functions

(i) $\frac{d}{dx}\log x = \frac{1}{x}$, for $x > 0$

(ii) $\frac{d}{dx}e^x = e^x$

(iii) $\frac{d}{dx}a^x = a^x \log a$, for $a > 0$

(iv) $\frac{d}{dx}\log_a x = \frac{1}{x \log a}$, for $x > 0$, $a > 0$, $a \neq 1$

(4) **Differentiation of inverse trigonometrical functions:** The following formulae can be applied directly while differentiating inverse trigonometrical functions

(i) $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

(ii) $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$, for $-1 < x < 1$

(iii) $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$

(iv) $\frac{d}{dx}\text{cosec}^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$, for $|x| > 1$

(v) $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$, for $x \in R$

(vi) $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$, for $x \in R$

(5) **Differentiation of hyperbolic functions:**

(i) $\frac{d}{dx}\sin h\, x = \cos h\, x$

(ii) $\frac{d}{dx}\cos h\, x = \sin h\, x$

(iii) $\frac{d}{dx}\tan h\, x = \sec h^2 x$

(iv) $\frac{d}{dx}\cot h\, x = -\text{cosec}\ h^2 x$

(v) $\frac{d}{dx}\sec h\, x = -\sec h\, x \tan h\ \ x$

(vi) $\frac{d}{dx}\text{cosec}\ h\, x = -\text{cosec}\ h\, x \cot h\, x$

(vii) $\frac{d}{dx}\sin h^{-1}x = 1/\sqrt{(1+x^2)}$

(viii) $\frac{d}{dx}\cos h^{-1}x = 1/\sqrt{(x^2-1)}$

(ix) $\frac{d}{dx}\tan h^{-1}x = 1/(x^2-1)$

(x) $\frac{d}{dx}\cot h^{-1}x = 1/(1-x^2)$

(xi) $\frac{d}{dx}\sec h^{-1}x = -1/x\sqrt{(1-x^2)}$

(xii) $\frac{d}{dx}\text{cosec}\ h^{-1}x = -1/x\sqrt{(1+x^2)}$

(6) **Differentiation by inverse trigonometrical substitution:** For trigonometrical substitutions following formulae and substitution should be remembered

(i) $\sin^{-1}x + \cos^{-1}x = \pi/2$

(ii) $\tan^{-1}x + \cot^{-1}x = \pi/2$

(iii) $\sec^{-1}x + co\sec^{-1}x = \pi/2$

(iv) $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right]$

(v) $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left[xy \mp \sqrt{(1-x^2)(1-y^2)}\right]$

(vi) $\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left[\dfrac{x \pm y}{1 \mp xy}\right]$

(vii) $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$

(viii) $2\cos^{-1} x = \cos^{-1}(2x^2 - 1)$

(ix) $2\tan^{-1} x = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

(x) $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$

(xi) $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$

(xii) $3\tan^{-1} x = \tan^{-1}\left(\dfrac{3x - x^3}{1-3x^2}\right)$

(xiii) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left(\dfrac{x+y+z-xyz}{1-xy-yz-zx}\right)$

(xiv) $\sin^{-1}(-x) = -\sin^{-1} x$

(xv) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

(xvi) $\tan^{-1}(-x) = -\tan^{-1} x$ or $\pi - \tan^{-1} x$

(xvii) $\dfrac{\pi}{4} - \tan^{-1} x = \tan^{-1}\left(\dfrac{1-x}{1+x}\right)$

(7) **Some suitable substitutions**

S. N.	Function	Substitution	S. N.	Function	Substitution
(i)	$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ or $a\cos\theta$	(ii)	$\sqrt{x^2 + a^2}$	$x = a\tan\theta$ or $a\cot\theta$
(iii)	$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$	(iv)	$\sqrt{\frac{a-x}{a+x}}$	$x = a\cos 2\theta$
(v)	$\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}$	$x^2 = a^2\cos 2\theta$	(vi)	$\sqrt{ax - x^2}$	$x = a\sin^2\theta$
(vii)	$\sqrt{\frac{x}{a+x}}$	$x = a\tan^2\theta$	(viii)	$\sqrt{\frac{x}{a-x}}$	$x = a\sin^2\theta$
(ix)	$\sqrt{(x-a)(x-}$	$x = a\sec^2\theta - b\tan^2\theta$	(x)	$\sqrt{(x-a)(b-}$	$x = a\cos^2\theta + b\sin$