

Relation between dy/dx and dx/dy .

Let x and y be two variables connected by a relation of the form $f(x, y) = 0$. Let Δx be a small change in x and let Δy be the corresponding change in y . Then $\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$ and $\frac{dx}{dy} = \lim_{\Delta y \rightarrow 0} \frac{\Delta x}{\Delta y}$.

$$\text{Now, } \frac{\Delta y}{\Delta x} \cdot \frac{\Delta x}{\Delta y} = 1 \Rightarrow \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta y}{\Delta x} \cdot \frac{\Delta x}{\Delta y} \right) = 1$$

$$\Rightarrow \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \cdot \lim_{\Delta y \rightarrow 0} \frac{\Delta x}{\Delta y} = 1 \quad [\because \Delta x \rightarrow 0 \Leftrightarrow \Delta y \rightarrow 0] \quad \Rightarrow \frac{dy}{dx} \cdot \frac{dx}{dy} = 1. \text{ So, } \frac{dy}{dx} = \frac{1}{dx / dy}.$$