Methods of Differentiation.

(1) **Differentiation of implicit functions:** If y is expressed entirely in terms of x, then we say that y is an explicit function of x. For example $y = \sin x$, $y = e^x$, $y = x^2 + x + 1$ etc. If y is related to x but cannot be conveniently expressed in the form of $y = f(x)$ but can be expressed in the form $f(x, y) = 0$, then we say that y is an implicit function of x.

(i) Working rule 1: (a) Differentiate each term of $f(x, y) = 0$ with respect to x.

(b) Collect the terms containing dy/dx on one side and the terms not involving dy/dx on the other side.

(c) Express dy/dx as a function of x or y or both.

Note: In case of implicit differentiation, dy/dx may contain both x and y.

(ii)Working rule 2: If $f(x, y)$ = constant, then $\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$

Where $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial x}$ are partial differential coefficients of $f(x, y)$ with respect to x and y respectively.

Note: Partial differential coefficient of $f(x, y)$ with respect to x means the ordinary differential coefficient of $f(x, y)$ with respect to x keeping y constant.

(3) **Differentiation of parametric functions:** Sometimes x and y are given as functions of a single variable, *e.g.*, $x = \phi(t), y = \psi(t)$ are two functions and t is a variable. In such a case x and y are called parametric functions or parametric equations and t is called the parameter. To find $\frac{dy}{dx}$ in case of parametric functions, we first obtain the relationship between x and y by eliminating the parameter t and then we differentiate it with respect to x. But every time it is not convenient to eliminate the parameter.

Therefore $\frac{dy}{dx}$ can also be obtained by the following formula

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

To prove it, let Δx and Δy be the changes in x and y respectively corresponding to a small change Δt in t.

Since $\frac{\Delta y}{\Delta x} = \frac{\Delta y / \Delta t}{\Delta x / \Delta t}$, $\quad \therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}}{\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\Psi'(t)}{\phi'(t)}$

(4) **Differentiation of infinite series:** If y is given in the form of infinite series of x and we have to find out $\frac{dy}{dx}$ then we remove one or more terms, it does not affect the series

(i) If $y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) + ... \infty}}}$, then $y = \sqrt{f(x) + y} \Rightarrow y^2 = f(x) + y$

$2y \frac{dy}{dx} = f'(x) + \frac{dy}{dx}$, $\quad \therefore \frac{dy}{dx} = \frac{f'(x)}{2y - 1}$

(ii) If $y = f(x)^{f(x)^{f(x)^{f(x)..... \infty}}}$ then $y = f(x)^y$

$\therefore \log y = y \log f(x)$

$\frac{1}{y} \frac{dy}{dx} = \frac{y.f'(x)}{f(x)} + \log f(x). \frac{dy}{dx}$, $\quad \therefore \frac{dy}{dx} = \frac{y^2 f'(x)}{f(x)[1 - y \log f(x)]}$

(iii) If $y = f(x) + \cfrac{1}{f(x) + \cfrac{1}{f(x) + ... \infty}}$ then $\frac{dy}{dx} = \frac{yf'(x)}{2y - f(x)}$

(5) **Differentiation of composite function:** Suppose function is given in form of $fog(x)$ or $f[g(x)]$

Working rule**:** Differentiate applying chain rule $\frac{d}{dx} f[g(x)] = f'[g(x)].g'(x)$