Successive Differentiation or Higher Order Derivatives.

(1) **Definition and notation:** If y is a function of x and is differentiable with respect to x, then its derivative $\frac{dy}{dx}$ can be found which is known as derivative of first order. If the first derivative $\frac{dy}{dx}$ is also a differentiable. Function, then it can be further differentiated with respect to x and this derivative is denoted by d^2y/dx^2 which is called the second derivative of y with respect to x further if $\frac{d^2y}{dx^2}$ is also differentiable then its derivative is called third derivative of y which is denoted by $\frac{d^3y}{dx^3}$. Similarly n^{th} derivative of y is denoted by $\frac{d^ny}{dx^n}$. All these derivatives are called as successive derivative and this process is known as successive differentiation. We also use the following symbols for the successive derivatives of $y = f(x)$:

$y_1,\ \ y_2,\ \ y_{3,}\ldots\ldots\ldots,\ y_n,\ldots\ldots$ $\qquad$ $y^I,\ \ y^{II},\ \ y^{III}\ldots\ldots\ldots,\ y^n,\ldots\ldots$

$Dy,\ \ D^2y,\ \ D^3y\ldots\ldots\ldots,\ D^ny,\ldots\ldots$ (Where $D = \frac{d}{dx}$) $\qquad$ $\frac{dy}{dx},\ \frac{d^2y}{dx^2},\ \frac{d^3y}{dx^3},\ldots\ldots\ \frac{d^ny}{dx^n},\ldots\ldots\ldots\ ..$

$f'(x),\ \ f''(x),\ \ f'''(x),\ldots\ldots\ .,f^n(x),\ldots\ldots$

If $y = f(x)$, then the value of the n^{th} order derivative at $x = a$ is usually denoted by

$\left(\frac{d^ny}{dx^n}\right)_{x=a}$ or $(y_n)_{x=a}$ or $(y^n)_{x=a}$ or $f^n(a)$

(2) n^{th} Derivatives of some standard functions:

(i) (a) $\frac{d^n}{dx^n}\sin(ax+b) = a^n\sin\left(\frac{n\pi}{2}+ax+b\right)$ $\qquad$ (b)

$\frac{d^n}{dx^n}\cos(ax+b) = a^n\cos\left(\frac{n\pi}{2}+ax+b\right)$

(ii) $\frac{d^n}{dx^n}(ax+b)^m = \frac{m!}{(m-n)!}a^n(ax+b)^{m-n}$, Where $m > n$

Particular cases:

(i)(a) When $m = n$

(ii) When $a = 1, b = 0$, then $y = x^n$

$D^n\{(ax+b)^n\} = a^n.n!$ $\therefore$

$D^n(x^m) = m(m-1).......(m-n+1)x^{m-n} = \frac{m!}{(m-n)!}x^{m-n}$

(b) When $m < n, D^n\{(ax+b)^m\} = 0$

(iii) When a = 1, b = 0 and m = n,

(iv)When $m = -1,\ y = \frac{1}{(ax+b)}$

Then $y = x^n$

$D^n(y) = a^n(-1)(-2)(-3)........(-n)(ax+b)^{-1-n}$

$\therefore D^n(x^n) = n!$

$= a^n(-1)^n(1.2.3......n)(ax+b)^{-1-n} = \frac{a^n(-1)^n n!}{(ax+b)^{n+1}}$

(3) $\frac{d^n}{dx^n}\log(ax+b) = \frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$

(4) $\frac{d^n}{dx^n}(e^{ax}) = a^n e^{ax}$

(5) $\frac{d^n(a^x)}{dx^n} = a^x(\log a)^n$

(6) (i) $\frac{d^n}{dx^n}e^{ax}\sin(bx+c) = r^n e^{ax}\sin(bx+c+n\phi)$

Where $r = \sqrt{a^2+b^2};\ \phi = \tan^{-1}\frac{b}{a},\ y = e^{ax}\sin(bx+c)$

(ii) $\frac{d^n}{dx^n}e^{ax}\cos(bx+c) = r^n e^{ax}\cos(bx+c+n\phi)$