Differentiation of Determinants.

Let
$$\Delta(x) = \begin{vmatrix} a_1(x) & b_1(x) \\ a_2(x) & b_2(x) \end{vmatrix}$$
. Then $\Delta'(x) = \begin{vmatrix} a'_1(x) & b'_1(x) \\ a_2(x) & b_2(x) \end{vmatrix} + \begin{vmatrix} a_1(x) & b_1(x) \\ a'_2(x) & b'_2(x) \end{vmatrix}$

If we write $\Delta(x) = |C_1 C_2 C_3|$. Then $\Delta'(x) = |C_1' C_2 C_3| + |C_1 C_2' C_3| + |C_1 C_2 C_3'|$

Similarly, if
$$\Delta(\mathbf{x}) = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix}$$
, then $\Delta'(\mathbf{x}) = \begin{vmatrix} R_1' \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2' \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R_3' \end{vmatrix}$

Thus, to differentiate a determinant, we differentiate one row (or column) at a time, keeping others unchanged.