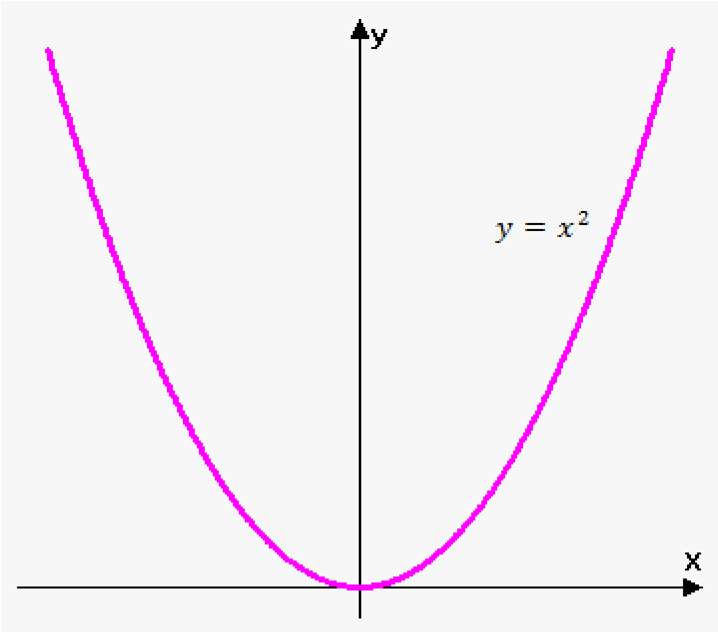
Increasing and Decreasing Function:



A function is said to be an increasing function if the value of y increases with the increase in x. As we can see from the above figure that at the right of the origin, the curve is going upward as we are going to the right so it is called **Increasing Function**.

A function is said to be a decreasing function if the value of y decreases with the increase in x. As above, in the left of the origin, the curve is going downward if we are moving from left to right.

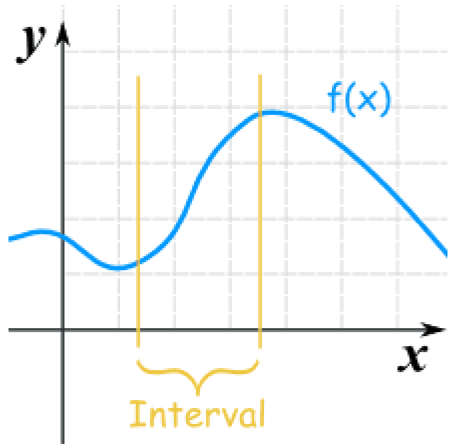
Definition of Increasing Function

This is the definition of a function which is increasing on an interval.

If there is a function **y = f(x)**

- A function is **increasing** over an interval, if for every x_1 and x_2 in the interval,

  $$x_1 < x_2,\ f(x_1) \le f(x_2)$$



- A function is **strictly increasing** over an interval, if for every x_1 and x_2 in the interval,

  $\mathbf{x_1 < x_2, f(x_1) < f(x_2)}$

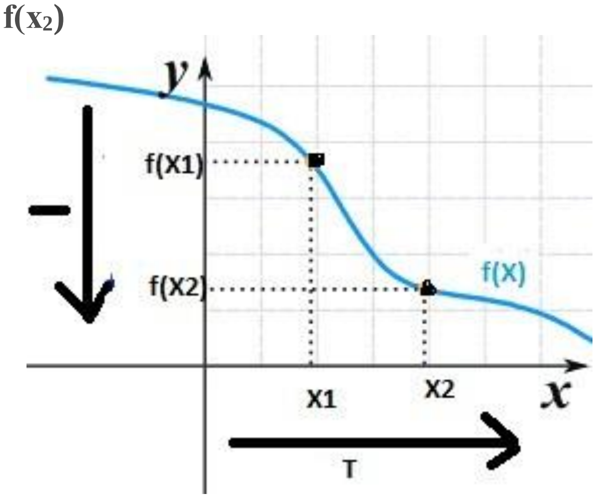
There is a difference of symbol in both the above increasing functions.

Definition of Decreasing Function

If there is a function **y = f(x)**

- A function is **decreasing** over an interval , if for every x_1 and x_2 in the interval,

  $\mathbf{x_1 < x_2, f(x_1) \geq f(x_2)}$



- A function is **strictly decreasing** over an interval, if for every x_1 and x_2 in the interval,

  $\mathbf{x_1 < x_2, f(x_1) > f(x_2)}$

There is a difference of symbol in both the above decreasing functions.

Definition of Increasing and Decreasing function at a point

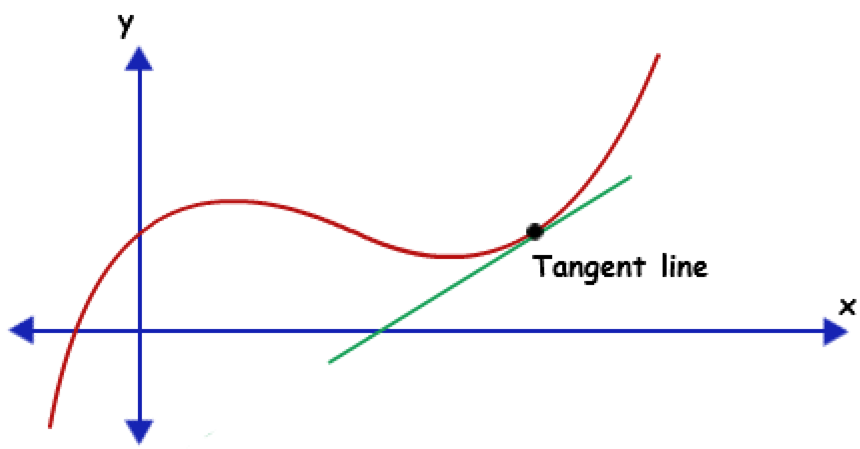
Let x_0 be a point on the curve of a real valued function f. Then f is said to be increasing, strictly increasing, decreasing or strictly decreasing at x_0, if there exists an open interval I containing x_0 such that f is increasing, strictly increasing, decreasing or strictly decreasing, respectively in I.

If there is a function f and interval I = (x_0 – h, x_0 + h), h > 0

- It is said to be **increasing** at x_0 if f is increasing in (x_o - h, x_0 + h)

  $x_1 < x_2$ in I $\Rightarrow f(x_1) \leq f(x_2)$
- It is said to be **strictly increasing** at x_0 if f is strictly increasing in (x_0 - h, x_0 + h)

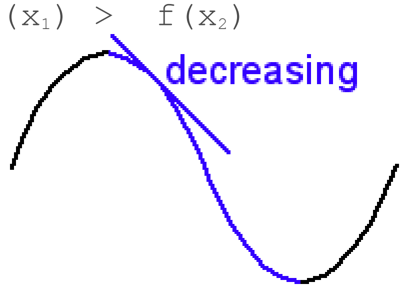
  $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$



- It is said to be **decreasing** at x_0 if f is decreasing in (x_o - h, x_0 + h)

  $x_1 < x_2$ in I $\Rightarrow f(x_1) \geq f(x_2)$
- It is said to be **strictly decreasing** at x_0 if f is strictly decreasing in (x_0 - h, x_0 + h)

  $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$



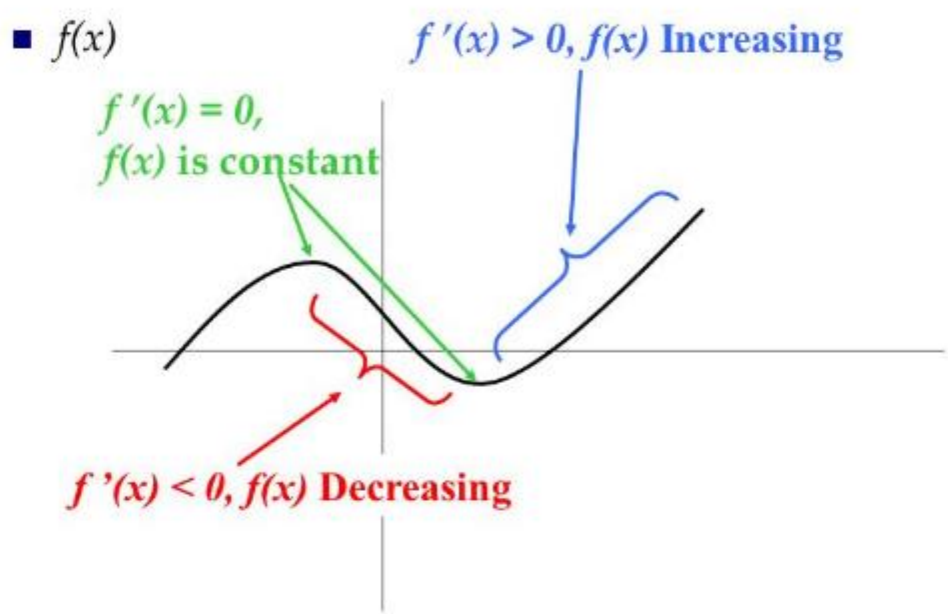
How derivatives are used to find whether the function is Increasing or Decreasing Function?

We can use the first derivative test to check whether the function is increasing or decreasing.

Theorem

Let f be continuous on [a, b] and differentiable on the open interval (a, b). Then

(a) If $f'(x) > 0$ for each $x \in (a, b)$ then f is **increasing** in interval [a, b]
(b) If $f'(x) < 0$ for each $x \in (a, b)$ then f is **decreasing** in interval [a, b]
(c) If $f'(x) = 0$ for each $x \in (a, b)$ then f is a **constant** function in [a, b]



This can be proved with the help of mean value theorem.

Proof:

Let $x_1, x_2 \in [a, b]$ such that $x_1 < x_2$
Now we can prove it with the help of Mean value theorem, which says that there is a point c between x_1 and x_2 so that
Let $x_1, x_2 \in [a, b]$ such that $x_1 < x_2$

Now we can prove it with the help of Mean value theorem, which says that there is a point c between x_1 and x_2 so that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

a. Let f'(c) ≥ 0

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} \geq 0$$

$$f(x_2) - f(x_1) \geq 0$$

$$f(x_2) \geq f(x_1)$$

Hence, f is an increasing function.

b. Let f'(c) ≤ 0

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} \leq 0$$

$$f(x_2) - f(x_1) \leq 0$$

$$f(x_2) \leq f(x_1)$$

Hence, f is a decreasing function.

c. Let f'(c) = 0

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = 0$$

$$f(x_2) - f(x_1) = 0$$

$$f(x_2) = f(x_1)$$

Hence, f is a constant function.

1. f is strictly increasing in (a, b) if f ' (x) > 0, ∀ x ∈ [a, b]

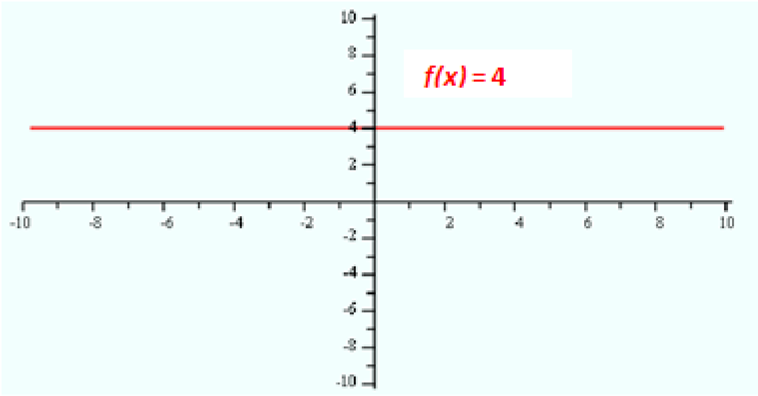
2. f is strictly decreasing in (a, b) if f ' (x) < 0, ∀ x ∈ [a, b]

3. f is increasing or decreasing on R if it is increasing or decreasing in every interval of R

What is constant function?

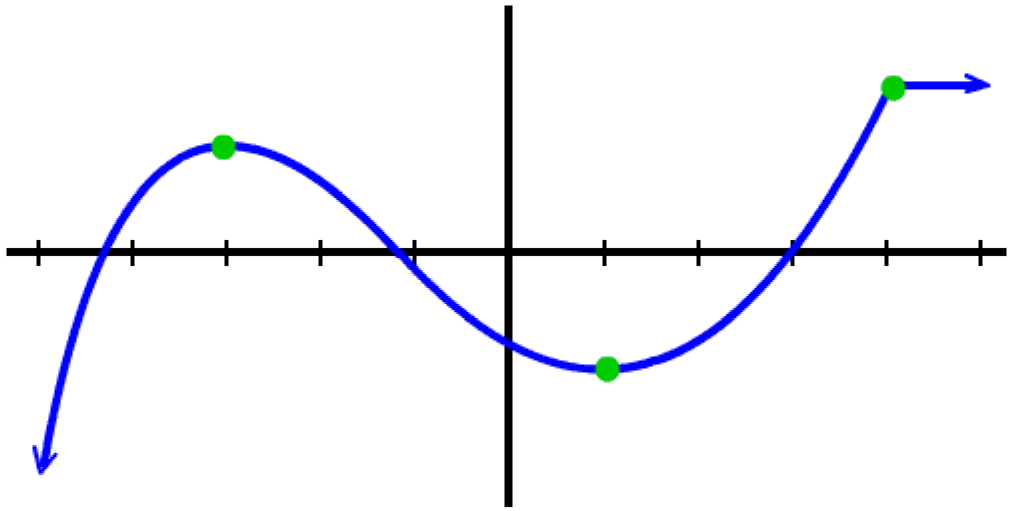
Constant function is a horizontal line.

A Constant Function Graph



As we know that the derivative is zero and y is always 4 in the above figure. This is the graph of line with one variable.

If we talk about curve, the function will be constant if its f'(c) = 0.

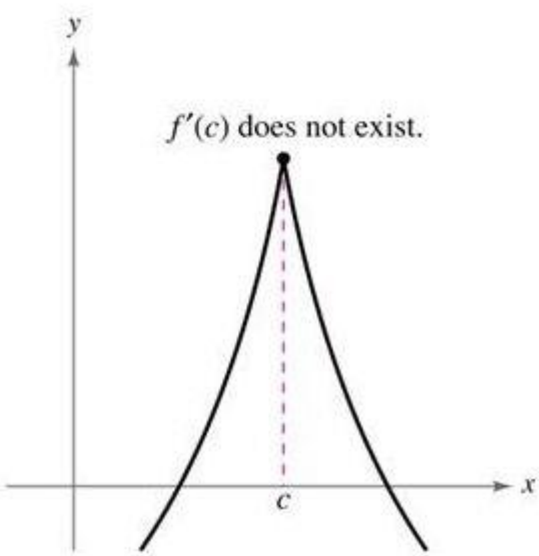


Here in the above figure, at green points the curve is neither increasing nor decreasing. The slope of the curve is zero at these points. It could be the highest or the lowest point of the curve in its neighborhood.

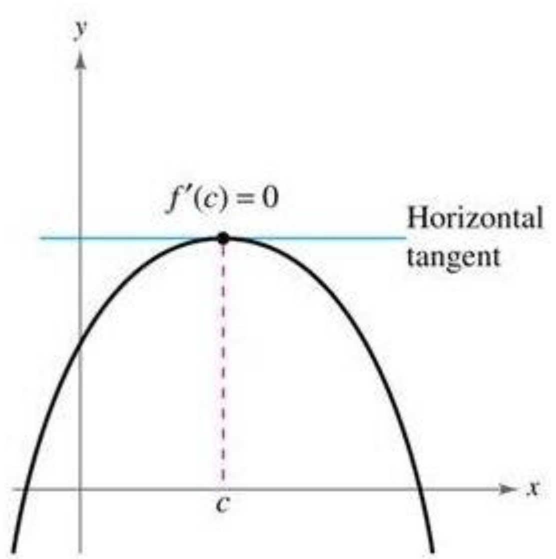
Definition of Critical Numbers

The **Critical Numbers** for a function f are those numbers c in the domain of f for which f'(c) = 0 or does not exists.

A **critical point** is a point whose x coordinate is the critical number c and the y coordinate is the f(c).



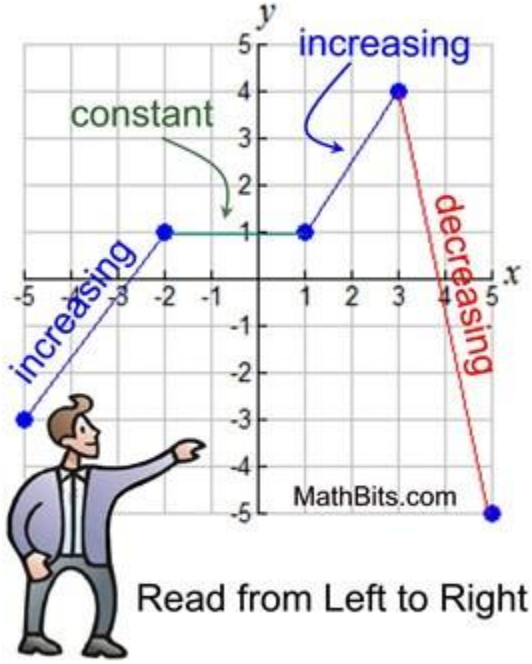
c is a critical number of f.



What are intervals of increase and decrease?

Interval is basically all the numbers between given two numbers.

If we talk about curve, we can say the portion of curve which is coming in between the two given numbers on the x-axis is the required interval.



As in the above figure,

Interval	Type of function
(-5,-2)	Increasing
(-2,1)	Constant
(1,3)	Increasing
(3,5)	decreasing

Calculation of intervals of increase or decrease

To calculate the intervals of increase or decrease function, we need to follow some steps:

- First of all, we have to differentiate the given function.
- Then solve the first derivative as equation to find the value of x.
    - The first derivative: $f'(x) = 0$.
- Form open intervals with the values of the x which we got after solving the first derivative and the points of discontinuity.
- Take a value from every interval and find the sign they have in the first derivative.
    - If $f'(x) > 0$ is increasing.
    - If $f'(x) < 0$ is decreasing.
- Write the intervals of increase and decrease: