Rolle's Theorem in calculus:

It is a special case of mean value theorem which says that if y = f (x) be a given function and satisfies the conditions:

(i) f (x) is continuous on the closed interval [a , b]

(ii) f (x) is differentiable on the open interval (a , b)

(iii) f (a) = f (b)

Then there must be one point c which comes between a and b so that a < c < b.

Hence f'(c) = 0 at least once for some x ∈ (a, b)

that is, the first derivative is zero or you can say that the slope of the tangent line to the curve of function is zero.

Note:

(1) There can be more than one such.

(2) Think! The conditions of Rolle's theorem are necessary or sufficient or both? The answer is conditions are only sufficient and necessary will be clear from the following examples:

(a) Let

$$f(X) = \frac{1}{X} + \frac{1}{1-X} \text{ if } 0 < X < 1$$
$$f(0) = f(1) = 0$$

Here condition (i) is violated

However f '(x) = 0 if x = 1/2 ∈ (0, 1)

By defining f(0) = 0 and f(1) = 3 we can see that the result is true when (i) and (iii) are violated.

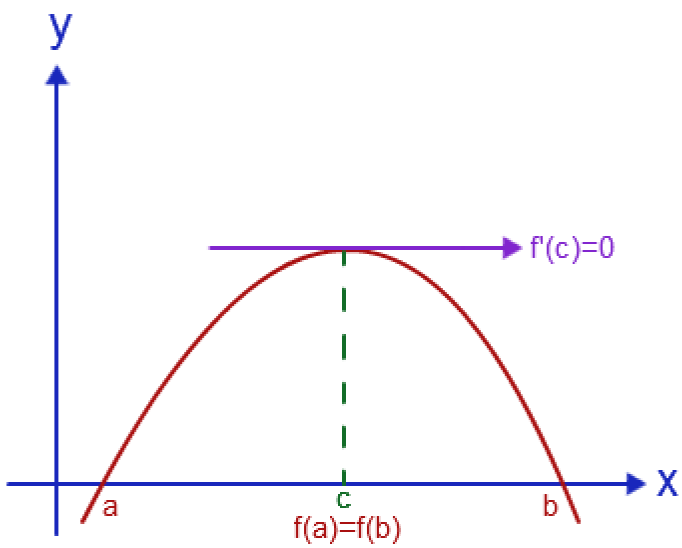
(b) Let

$$f(X) = \begin{cases} X, & 0 \le x \le 1 \\ 1, & 1 \le x < 2 \end{cases}$$

Clearly (ii) does not hold in (0, 2) and even so f'(3/2) = 0

(3) If f(x) is any polynomial then between any pair of roots of f (x) = 0 lies a root of f ’(x) = 0.

Graphical representation of the Rolle’s Theorem



Here the graph of function **y = f(x) is continuous** between **x = a and x = b**. there could be so many tangents but at least one tangent is there which is parallel to x axis. At that point **f'(c) = 0.**

Example 1:

If $ax^2 + bx + c = 0$, a, b, c $\in$ **R. Find the condition that this equation would have at least one root in (0, 1)**.

Solution: Let $f'(x) = ax^2 + bx + c$

Integrating both sides,

=> $f(x) = ax^3 / 3 + bx^2 / 2 + cx + d$

=> f(0) = d and f(1) = a/3 + b/2 + c + d

Since, Rolle’s theorem is applicable

=> $f(0) = f(1)$ => $d = a/3 + b/2 + c + d$

=> $2a + 3b + 6c = 0$

Hence required condition is $2a + 3b + 6c = 0$