

Some Important results of Definite Integral.

(1) If $I_n = \int_0^{\pi/4} \tan^n x dx$ then $I_n + I_{n-2} = \frac{1}{n-1}$

(2) If $I_n = \int_0^{\pi/4} \cot^n x dx$ then $I_n + I_{n-2} = \frac{1}{1-n}$

(3) If $I_n = \int_0^{\pi/4} \sec^n x dx$ then $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$

(4) If $I_n = \int_0^{\pi/4} \operatorname{cosec}^n x dx$ then $I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$

(5) If $I_n = \int_0^{\pi/2} x^n \sin x dx$ then $I_n + n(n-1)I_{n-2} = n(\pi/2)^{n-1}$

(6) If $I_n = \int_0^{\pi/2} x^n \cos x dx$ then $I_n + n(n-1)I_{n-2} = (\pi/2)^n$

(7) If $a > b > 0$, then $\int_0^{\pi/2} \frac{dx}{a+b \cos x} = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1} \sqrt{\frac{a+b}{a-b}}$

(8) If $0 < a < b$ then $\int_0^{\pi/2} \frac{dx}{a+b \cos x} = \frac{1}{\sqrt{b^2-a^2}} \log \left| \frac{\sqrt{b+a} - \sqrt{b-a}}{\sqrt{b+a} + \sqrt{b-a}} \right|$

(9) If $a > b > 0$ then $\int_0^{\pi/2} \frac{dx}{a+b \sin x} = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1} \sqrt{\frac{a-b}{a+b}}$

(10) If $0 < a < b$, then $\int_0^{\pi/2} \frac{dx}{a+b \sin x} = \frac{1}{\sqrt{b^2-a^2}} \log \left| \frac{\sqrt{b+a} + \sqrt{b-a}}{\sqrt{b+a} - \sqrt{b-a}} \right|$

(11) If $a > b, a^2 > b^2 + c^2$, then $\int_0^{\pi/2} \frac{dx}{a+b \cos x + c \sin x} = \frac{2}{\sqrt{a^2-b^2-c^2}} \tan^{-1} \frac{a-b+c}{\sqrt{a^2-b^2-c^2}}$

(12) If $a > b, a^2 < b^2 + c^2$, then

$$\int_0^{\pi/2} \frac{dx}{a + b \cos x + c \sin x} = \frac{1}{\sqrt{b^2 + c^2 - a^2}} \log \left| \frac{a - b + c - \sqrt{b^2 + c^2 - a^2}}{a - b + c + \sqrt{b^2 + c^2 - a^2}} \right|$$

(13) If $a < b, a^2 < b^2 + c^2$ then

$$\int_0^{\pi/2} \frac{dx}{a + b \cos x + c \sin x} = \frac{-1}{\sqrt{b^2 + c^2 - a^2}} \log \left| \frac{b - a - c - \sqrt{b^2 + c^2 - a^2}}{b - a - c + \sqrt{b^2 + c^2 - a^2}} \right|$$

Important Tips

$$\Rightarrow \lim_{x \rightarrow 0} \left| \frac{\int_0^x f(x) dx}{x} \right| = f(0)$$

$$\Rightarrow \int_a^b f(x) dx = (b - a) \int_0^1 f[(b - a)t + a] dt$$