

Evaluation of Definite Integral by Substitution.

When the variable in a definite integral is changed, the substitutions in terms of new variable should be effected at three places.

(i) In the integrand

(ii) In the differential say, dx

(iii) In the limits

For example, if we put $\phi(x) = t$ in the integral $\int_a^b f\{\phi(x)\}\phi'(x)dx$, Then $\int_a^b f\{\phi(x)\}\phi'(x)dx = \int_{\phi(a)}^{\phi(b)} f(t)dt$.

Important Tips

$$\int_0^\pi \frac{dx}{1 + \sin x} = 2$$

$$\int_0^{\pi/2} \log(\tan x) dx = 0$$

$$\int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \frac{\pi}{2}$$

$$\int_0^a \sqrt{a^2 - x^2} dx = \frac{\pi a^2}{4}$$

$$\int_0^{\pi/2} \frac{dx}{\sin x + \cos x} = \sqrt{2} \log(\sqrt{2} + 1)$$

$$\int_0^a \frac{dx}{1 + e^{f(x)}} = \frac{a}{2}, \text{ where } f(a-x) = -f(x)$$

$$\int_0^a \frac{dx}{x^2 + a^2} = \frac{\pi}{2a}$$