

1. Summation of Series by Integration.

We know that $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} h \sum_{r=1}^n f(a + rh)$, where $nh = b - a$

Now, put $a = 0, b = 1, \therefore nh = 1$ or $h = \frac{1}{n}$. Hence $\int_0^1 f(x)dx = \lim_{n \rightarrow \infty} \frac{1}{n} \sum f\left(\frac{r}{n}\right)$

Note: Express the given series in the form $\sum \frac{1}{n} f\left(\frac{r}{h}\right)$. Replace $\frac{r}{n}$ by x , $\frac{1}{n}$ by dx and the limit of the sum is

$$\int_0^1 f(x)dx .$$