Mathematical Reasoning:

- A sentence is called a mathematically acceptable statement if it is either true or false but not both.
- A sentence is neither imperative nor interrogative nor exclamatory.
- A declarative sentence containing variables is an open statement if it becomes a statement when the variables are replaced by some definite values.
- A compound statement is a statement which is made up of two or more statements. Each of this statement is termed to be a compound statement.
- The compound statements are combined by the word “and” (^) the resulting statement is called a conjunction denoted as $p \wedge q$.
- The compound statement with “And” is true if all its component statements are true.
- The following truth table shows the truth values of $p \wedge q$ (p and q) and $q \wedge p$ (q and p):

Truth Table ($p \vee q$, $q \vee p$)			
p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F
Rule: $p \wedge q$ is true only when p and q are true			

- Compound statements p and q are combined by the connective ‘OR’

(v) then the compound statement denoted as p v q so formed is called a disjunction.??

Truth Table (p v q, q v p)			
p	q	p v q	q v p
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F
Rule: p v q is false only when both p and q are false.			

- The denial of a statement is called the negation of the statement. The truth table for the same is given below:

Truth Table (~p)		
p	~p	~ (~p)
T	F	T
F	T	F
Rule: ~ is true only when p is false		

- Negation is not a binary operation, it is a unary operation i.e. a modifier.
- There are three types of implications:
- "If ……. then"
- "Only if"
- "If and only if"
- "If …. then" type of compound statement is called conditional statement. The statement 'if p then q' is denoted by p → q or by p ⇒

q. p → q also means:

- p is sufficient for q
- q is necessary for p
- p only if q
- p leads to q
- q if p
- q when p
- if p then q
- Truth table for p → q
-

Truth Table (p → q, q → p)			
p	q	p → q	q → p
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T
Rule: p → q is false only when p is true and q is false.			

- If and only if type of compound statement is called biconditional or equivalence or double conditional. represnted as p ⇔ q or p ↔ q, it means
- p is a necessary and sufficient condition for q
- q is a necessary and sufficient condition for p

q. $p \rightarrow q$ also means:

- p is sufficient for q
- q is necessary for p
- p only if q
- p leads to q
- q if p
- q when p
- if p then q
- Truth table for $p \rightarrow q$
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Truth Table ($p \rightarrow q$, $q \rightarrow p$)			
p	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T
Rule: $p \rightarrow q$ is false only when p is true and q is false.			

- If and only if type of compound statement is called biconditional or equivalence or double conditional. represnted as $p \Leftrightarrow q$ or $p \leftrightarrow q$, it means
- p is a necessary and sufficient condition for q
- q is a necessary and sufficient condition for p

- If p then q and if q then p
- q if and only if p
- Truth table for p ↔ q or q ↔ p

Truth Table (p ↔ q, q ↔ p)			
p	q	p ↔ q	q ↔ p
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T
Rule: p ↔ q is true only when both p and q have the same truth value.			

- Contrapositive of p → q is ∼ q → ∼ p.
- Converse of p → q is q → p.
- Truth table for p → q

Truth Table (p → q)				
p	q	p → q	~q → ~p (Contrapositive)	q ↔ p (Converse)
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

- The compound statements which are true for any truth value of their components are called tautologies.
- Truth table for a tautology 'p ∨ ~p"?, p being a logical statement

Truth Table (p ∨ ~p)

p	~p	p ∨ ~p
T	F	T
F	T	T

- ?The negation of tautology is a fallacy or a contradiction. The truth table for ‘p ∧ ~ p” which is fallacy?, p being a logical statement is given below

Truth Table (p ∧ ~p)		
p	~p	p ∧ ~p
T	F	F
F	T	F

- **Important points on tautology and fallacy:**
- p ∨ q is true iff at least one of p and q is true
- p ∧ q is true iff both p and q are true
- A tautology is always true
- A fallacy is always false.
- **Statements satisfy the following laws:**
- **Idempotent Laws:** If p is any statement then p ∨ p = p and p ∧ p = p
- **Associative Laws:** If p, q, r are any three statements, then p ∨ (q ∨ r) = (p ∨ q) ∨ r and p ∧ (q ∧ r) = (p ∧ q) ∧ r
- **Commutative Laws:** If p, q are any two statements, then p ∨ q = q ∨ p and p ∧ q = q ∧ p
- **Distributive Laws:** If p, q, r are any three statements then p ∧ (q ∨

r) = (p ∧ q) ∨ (p ∧ r) and p ∨ (q ∧ r) = (p ∨ q) ∧ (p ∨ r)

- **Identity Laws:** If p is any statement, t is tautology and c is a contradiction, then $p \vee t = t$, $p \wedge t = p$, $p \vee c = p$ and $p \wedge c = c$
- **Complement Laws:** If t is tautology, c is a contradiction and p is any statement then $p \vee (\sim p) = t$, $p \wedge (\sim p) = c$, $\sim t = c$ and $\sim c = t$
- **Involution Law:** If p is any statement, then $\sim(\sim p) = p$
- **De-Morgan's Law:** If p and q are two statements then $\sim(p \vee q) = (\sim p) \wedge (\sim q)$ and $\sim(p \wedge q) = (\sim p) \vee (\sim q)$