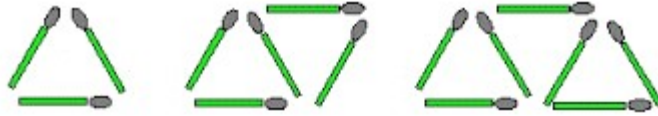


Meaning of Sequence

A sequence is a group of objects which follow some particular pattern. If we have some objects listed in some order so that it has 1st term, 2nd term and so on, then it is a sequence.

Example



In this image we can see the group of sticks with a certain pattern, so we can observe that what could come next

As we can see that the next figure in the sequence will have 9 sticks.

What is a sequence in Math?

In Mathematics, it is a group of numbers in an ordered form which follow a certain pattern is called **Sequence**.

$$5, 8, 11, 14, 17, \dots$$

Sequence A

These numbers are called '**Term**' or '**Members**' or '**Elements**'. It is similar as a set of numbers.

As in the above figure 5,8,11,14,17 are the terms of the sequence.

Finite Sequence

The sequence which have limited or finite number of terms is called **Finite Sequence**.

Example

{1, 3, 9, 27} is the sequence of multiples of 3.

{m, o, n, k, e, y} is the sequence of letters in the word "monkey".

Infinite Sequence

The sequence which have unlimited or infinite number of terms or it has no end is called **Infinite Sequence**.

Example

2, 4, 6, 8, ...

This is the infinite sequence of even numbers. It is the three dots here which shows that it is an infinite sequence, with no end. So its last term will be represented by n_{∞} .

In the other way we can write it as

$$\{a_n\}_{n=1}^{\infty}$$

Here ∞ the infinity means infinite sequence.

Types of Sequence

There are three types of sequence

- Arithmetic Sequence
- Geometric Sequence
- Fibonacci

Sequence

Arithmetic Sequence

Any sequence in which the difference between every successive terms is constant then it is called **Arithmetic Sequence**. It could be in ascending or descending form according to the **constant number**.

Example

0, 3, 6, 9, 12, ...
↓ ↓ ↓ ↓
+3 +3 +3 +3

Here we are getting the terms by adding 3 every time. this is the difference between the two successive terms so it is called the difference.

The difference is represented by “d”.

In the above example we can see that $a_1 = 0$ and $a_2 = 3$.

The difference between the two successive terms is

$$a_2 - a_1 = 3$$

$$a_3 - a_2 = 3$$

If the first term of an arithmetic sequence is a_1 and the common difference is d , then the n th term of the sequence is given by:

$$a_n = a_1 + (n-1)d$$

Geometric Sequence

Any sequence in which the ratio between every successive terms is constant then it is called **Geometric Sequence**. It could be in ascending or descending form according to the **constant ratio**.

Example

1, 4, 16, 64, ...

Here

$$a_1 = 1$$

$$a_2 = 4 = a_1(4)$$

$$a_3 = 16 = a_2(4)$$

Here we are multiplying it with 4 every time to get the next term. Here the ratio is 4 .

The ratio is denoted by “r”.

$$a_n = a_{n-1} \cdot r \quad \text{or} \quad a_n = a_1 \cdot r^{n-1}$$

Fibonacci Sequence

By adding the value of the two terms before the required term, we will get the next term. Such type of sequence is called Fibonacci sequence. There is no visible pattern.

Example

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

In the above sequence, we can see

$$a_1 = 0 \text{ and } a_2 = 1$$

$$a_3 = a_2 + a_1 = 0 + 1 = 1$$

$$a_4 = a_3 + a_2 = 1 + 1 = 2 \text{ and so on.}$$

So the formula of the Fibonacci Sequence is

$$a_n = a_{n-2} + a_{n-1}, n > 2$$

This is also called the **Recursive Formula**.

Meaning of Series

The summation of all the numbers of the sequence is called **Series**. Generally it is written as S_n .

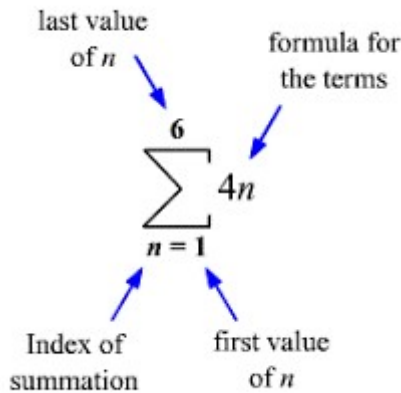
Example

If we have a sequence 1, 4, 7, 10, ...

Then the series of this sequence is $1 + 4 + 7 + 10 + \dots$

Notation of Series

We use the sigma notation that is, **the Greek symbol “ Σ ”** for the series which means “**sum up**”.



The series $4 + 8 + 12 + 16 + 20 + 24$ can be expressed as $\sum_{n=1}^6 4n$. We read this expression as the sum of $4n$ as n goes from 1 to 6.

Finite and Infinite Series

A series with finite number of terms is called **Finite Series**.

$$2 + 4 + 6 + 8 + 10$$

A series infinite number of terms is called **Infinite Series**.

$$2 + 4 + 6 + 8 + 10 + \dots$$

Types of Series

There are different types of series-

Arithmetic Series

Arithmetic series is the summation of the terms of the arithmetic sequence that is, if the difference between the every term to its preceding term is always constant then it is said to be an

The arithmetic series is in the form of

$$\{a + (a + d) + (a + 2d) + (a + 3d) + \dots\}$$

where a is the first term of the series and d is the difference of it which is known as the common difference of the given series.

Formula of Arithmetic Series

If a is the first term, d is the difference and n is the total number of the terms, then the formula for n th term is given by

$$a_n = a + (n - 1) d$$

Sum of an Arithmetic Series

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Geometric Series

Geometric series is the summation of the terms of the geometric sequence i.e. if the ratio between the every term to its preceding term is always constant then it is said to be a geometric series.

Formula of Geometric Series

In general, we can define geometric series as

$$\sum_{n=1}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \dots + ar^n$$

Where, a is the first term of the series and r is the common ratio for it.

Formula for nth term of the geometric series

$$a_n = a \cdot r^{n-1}$$

Where, n is the number of the term.

Sum of Geometric Series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

Meaning of Arithmetic Progression (A.P.)

Arithmetic Progression is the sequence of numbers such that the difference between the two successive terms is always constant. And that difference is called the common difference. It is also known as **Arithmetic Sequence**.

Meaning of Geometric Progression (G.P.)

Geometric Progression is the sequence of numbers such that the next term of the sequence comes by multiplying or dividing the preceding number with the constant (non-zero) number. And that constant number is called the **Common Ratio**. It is also known as **Geometric Sequence**.

$$a, ar, ar^2, ar^3, \dots, ar^n$$

Arithmetic Mean

Arithmetic mean is basically the average of two numbers. If we have two numbers n and m, then we can include a number L in between these numbers so that the three numbers will form an arithmetic sequence like n, L, m.

In that case the number L is the arithmetic mean of the numbers n and m.

According to the property of Arithmetic progression, we can say that-

$L - n = m - L$ that is, the common difference of the given AP.

$$L = \frac{n + m}{2}$$

This is generally used to find the missing number of the sequence between the two given numbers.

Example

What will be the 6th number of the sequence if the 5th term is 12 and the 7th term is 24?

As the two numbers are given so the 6th number will be the Arithmetic mean of the two given numbers.

$$\begin{aligned} \text{AM} &= \frac{12 + 24}{2} \\ &= \frac{36}{2} \\ &= 18 \end{aligned}$$

Hence the 6th term will be 18.

Geometric Mean

Geometric Mean is the Special type of average of two numbers. If a and b are the two numbers then the geometric mean will be

$$\text{GM} = \sqrt{ab}$$

Example

Find the geometric mean of 2 and 18.

Solution:

We can use the above formula to calculate the geometric mean.

$a = 2$ and $b = 18$

$$\begin{aligned} \text{GM} &= \sqrt{ab} \\ &= \sqrt{2 \times 18} \\ &= 6 \end{aligned}$$

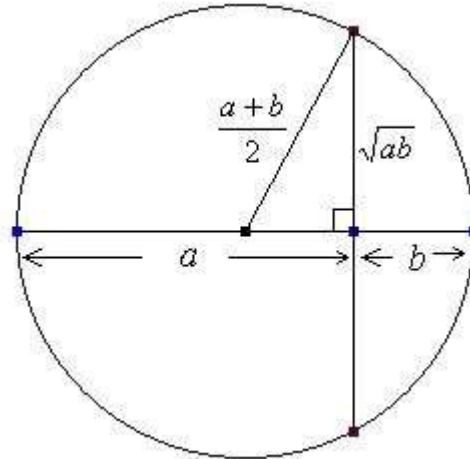
Relation between A.M. and G.M.

Here we can see that the sequence 2, 6, 18 is a geometric progression.

As we have seen above the formula for the Arithmetic mean and the Geometric mean are as follows:

$$\text{A. M.} = \frac{a+b}{2} \text{ and GM} = \sqrt{ab}$$

where a and b are the two given positive numbers.



Let A and G be A.M. and G.M.

So

$$A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}$$

Now let's subtract the two means with each other

$$\begin{aligned} A - G &= \frac{a+b}{2} - \sqrt{ab} \\ &= \frac{a+b-2\sqrt{ab}}{2} \\ &= \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0 \end{aligned}$$

This shows that $A \geq G$

Example

Find the two numbers, If Arithmetic mean and Geometric mean of two positive real numbers are 20 and 16, respectively.

Solution:

Given

$$\text{A.M} = \frac{a + b}{2} = 20$$

$$= a + b = 20 \quad \dots (1)$$

$$\text{G.M.} = \sqrt{ab} = 16$$

$$= ab = 256 \quad \dots (2)$$

Now we will put these values of a and b in

$$(a - b)^2 = (a + b)^2 - 4ab$$

$$(a - b)^2 = (40)^2 - 4(256)$$

$$= 1600 - 1024$$

$$= 576$$

$$a - b = \pm 24 \text{ (by taking the square root) } \dots (3)$$

By solving (1) and (3), we get

$$a + b = 40$$

$$a - b = 24$$

$$a = 8, b = 32 \text{ or } a = 32, b = 8$$

Special Series

Special Series are the series which are special in some way. It could be arithmetic or geometric.

Some of the special series are:

(i) $1 + 2 + 3 + \dots + n$ (sum of first n natural numbers)

(ii) $1^2 + 2^2 + 3^2 + \dots + n^2$ (sum of squares of the first n natural numbers)

(iii) $1^3 + 2^3 + 3^3 + \dots + n^3$ (sum of cubes of the first n natural numbers).

Sum to n terms of Special Series

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$