Gravitational Potential.

At a point in a gravitational field potential V is defined as negative of work done per unit mass in shifting a test mass from some reference point (usually at infinity) to the given point i.e.

$$V = -\frac{W}{m} = -\int \frac{\vec{F}.\vec{dr}}{m} = -\int \vec{I}.\vec{dr}$$ [As $\frac{F}{m} = I$]

$\therefore$ $$I = -\frac{dV}{dr}$$

i.e., negative gradient of potential gives intensity of field or potential is a scalar function of position whose space derivative gives intensity. Negative sign indicates that the direction of intensity is in the direction where the potential decreases.

Important points

(i) It is a scalar quantity because it is defined as work done per unit mass.

(ii) Unit: Joule/kg or m2/sec2

(iii) Dimension: [M0L2T–2]

(iv) If the field is produced by a point mass then $V = -\int I\, dr = -\int \left(-\frac{GM}{r^2}\right) dr$ [As $I = -\frac{GM}{r^2}$]

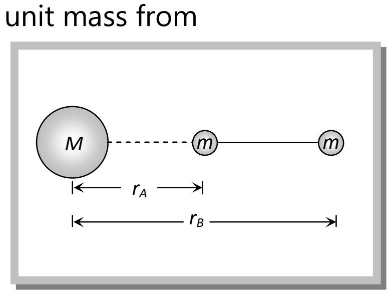
$\therefore$ $V = -\frac{GM}{r} + c$ [Here c = constant of integration]

Assuming reference point at ∞ and potential to be zero there we get

$$0 = -\frac{GM}{\infty} + c \Rightarrow c = 0$$

$\therefore$ Gravitational potential $V = -\frac{GM}{r}$

(v) Gravitational potential difference: It is defined as the work done to move a unit mass from



one point to the other in the gravitational field. The gravitational potential difference in bringing unit test mass m from point A to point B under the gravitational influence of source mass M is

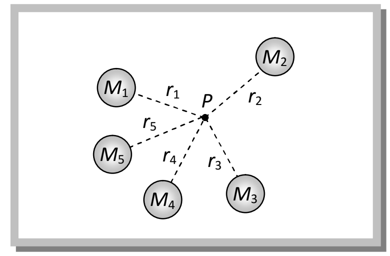
$$\Delta V = V_B - V_A = \frac{W_{A\to B}}{m} = -GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

(vi) Potential due to large numbers of particle is given by scalar addition of all the potentials.

$$V = V_1 + V_2 + V_3 + \dots\dots\dots$$

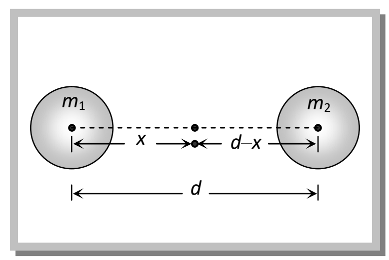
$$= -\frac{GM}{r_1} - \frac{GM}{r_2} - \frac{GM}{r_3}\dots\dots\dots$$

$$= -G\sum_{i=1}^{i=n}\frac{M_i}{r_i}$$



(vii) Point of zero potential: It is that point in the gravitational field, if the unit mass is shifted from infinity to that point then net work done will be equal to zero.

Let m1 and m2 are two masses placed at d distance apart and P is the point of zero potential in between the two masses.



Net potential for point $P = V_A + V_B = 0$

$$\Rightarrow \quad -\frac{Gm_1}{x} - \frac{Gm_2}{d-x} = 0$$

By solving $x = \dfrac{m_1 d}{m_1 - m_2}$