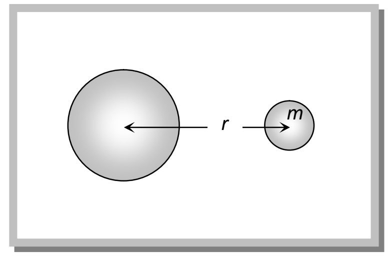
Gravitational Potential Energy.

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.



$$W = \int_{\infty}^{r} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational potential energy

$$\therefore U = -\frac{GMm}{r}$$

Important points

(i) Potential energy is a scalar quantity.

(ii) Unit: Joule

(iii) Dimension: [ML2T–2]

(iv) Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

(v) As the distance r increases, the gravitational potential energy becomes less negative i.e., it increases.

(vi) If $r = \infty$ then it becomes zero (maximum)

(vii) In case of discrete distribution of masses

Gravitational potential energy $U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$

(viii) If the body of mass m is moved from a point at a distance r_1 to a point at distance $r_2 (r_1 > r_2)$ then change in potential energy $\Delta U = \int_{r_1}^{r_2} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$ or

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

As r_1 is greater than r_2, the change in potential energy of the body will be negative. It means that if a body is brought closer to earth its potential energy decreases.

(ix) Relation between gravitational potential energy and potential $U = -\frac{GMm}{r} = m\left[\frac{-GM}{r}\right]$

$\therefore$ $U = mV$

(x) Gravitational potential energy at the center of earth relative to infinity.

$$U_{centre} = m\,V_{centre} = m\left(-\frac{3}{2}\frac{GM}{R}\right) = -\frac{3}{2}\frac{GMm}{R}$$

(xi) Gravitational potential energy of a body at height h from the earth surface is given by

$$U_h = -\frac{GMm}{R+h} = -\frac{gR^2m}{R+h} \equiv -\frac{mgR}{1+\frac{h}{R}}$$