Work Done Against Gravity.

If the body of mess m is moved from the surface of earth to a point at distance h above the surface of earth, then change in potential energy or work done against gravity will be

$$W = \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$\Rightarrow$ $$W = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$ [As $r_1 = R$ and $r_2 = R + h$]

$\Rightarrow$ $$W = \frac{GMmh}{R^2\left(1 + \frac{h}{R}\right)} = \frac{mgh}{1 + \frac{h}{R}}$$ [As $\frac{GM}{R^2} = g$]

Important points

(i) When the distance h is not negligible and is comparable to radius of the earth, then we will use above formula.

(ii) If $h = nR$ then $W = mgR\left(\frac{n}{n+1}\right)$

(iii) If $h = R$ then $W = \frac{1}{2}mgR$

(iv) If h is very small as compared to radius of the earth then term h/R can be neglected

From $$W = \frac{mgh}{1 + h/R} = mgh$$ $\left[\text{As } \frac{h}{R} \to 0\right]$