Time Period of Satellite.

It is the time taken by satellite to go once around the earth.

$$\therefore \quad T = \frac{\text{Circumference of the orbit}}{\text{orbital velocity}}$$

$$\Rightarrow \quad T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}} \qquad \left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{r^3}{gR^2}} \qquad [\text{As } GM = gR^2]$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}\left(1+\frac{h}{R}\right)^{3/2} \qquad [\text{As } r = R+h]$$

Important points

(i) From $T = 2\pi\sqrt{\frac{r^3}{GM}}$ it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit

(ii) $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\Rightarrow \quad T^2 = \frac{4\pi^2}{GM}r^3$ i.e., $T^2 \propto r^3$

This is in accordance with Kepler's third law of planetary motion r becomes a (semi major axis) if the orbit is elliptic.

(iii) Time period of nearby satellite,

From $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{gR^2}} = 2\pi\sqrt{\frac{R}{g}}$ [As $h = 0$ and $GM = gR^2$]

For earth $R = 6400\ km$ and $g = 9.8 m/s^2$

$$T = 84.6 \text{ minute} \approx 1.4\ hr$$

(iv) Time period of nearby satellite in terms of density of planet can be given as

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{GM}} = \frac{2\pi\left(R^3\right)^{1/2}}{\left[G.\frac{4}{3}\pi R^3\rho\right]^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$$

(v) If the gravitational force of attraction of the sun on the planet varies as $F \propto \frac{1}{r^n}$ then the time period varies as $T \propto r^{\frac{n+1}{2}}$

(vi) If there is a satellite in the equatorial plane rotating in the direction of earth's rotation from west to east, then for an observer, on the earth, angular velocity of satellite will be $(\omega_S - \omega_E)$. The time interval between the two consecutive appearances overhead will be

$$T = \frac{2\pi}{\omega_s - \omega_E} = \frac{T_S T_E}{T_E - T_S} \qquad \left[\text{As } T = \frac{2\pi}{\omega}\right]$$

If $\omega_S = \omega_E$, $T = \infty$ i.e. satellite will appear stationary relative to earth. Such satellites are called geostationary satellites.