Variation in g with Height.

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} \qquad \dots \dots (i)$$

Acceleration due to gravity at height h from the surface of the earth

$$g' = \frac{GM}{(R+h)^2} \qquad \dots (ii)$$

From (i) and (ii)
$$g' = g \left(\frac{R}{R+h}\right)^2 \qquad \dots (iii)$$
$$= \frac{g \frac{R^2}{r^2}}{r^2} \qquad \dots (iv) \qquad [As r = R + h]$$



Important points

(i) As we go above the surface of the earth, the value of g decreases because $g' \propto \frac{1}{r^2}$.

(ii) If $r = \infty$ then g' = 0, i.e., at infinite distance from the earth, the value of g becomes zero.

(iii) If $h \ll R$ i.e., height is negligible in comparison to the radius then from equation (iii) we get

$$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(1+\frac{h}{R}\right)^{-2} = g\left[1-\frac{2h}{R}\right]$$
 [As $h \ll R$]

(iv) If $h \ll R$ then decrease in the value of g with height:

Absolute decrease $\Delta g = g - g' = \frac{2hg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g - g'}{g} = \frac{2h}{R}$

Percentage decrease
$$\frac{\Delta g}{g} \times 100\% = \frac{2h}{R} \times 100\%$$