Young's Modulus (Y).

It is defined as the ratio of normal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{Al}$$

If force is applied on a wire of radius r by hanging a weight of mass M, then

$$Y = \frac{MgL}{\pi r^2 l}$$

Important points

(i) If the length of a wire is doubled,

Then longitudinal strain = $\frac{\text{change in length}(l)}{\text{initial length}(L)} = \frac{\text{final length} - \text{initial length}}{\text{Initial length}} = \frac{2L - L}{L} = 1$

$\therefore$ Young's modulus = $\frac{\text{stress}}{\text{strain}} \Rightarrow$ Y = stress [As strain = 1]

So young's modulus is numerically equal to the stress which will double the length of a wire.

(ii) Increment in the length of wire $l = \frac{FL}{\pi r^2 Y}$ $\left[\text{As } Y = \frac{FL}{Al}\right]$

So if same stretching force is applied to different wires of same material, $l \propto \frac{L}{r^2}$ [As F and Y are constant]

i.e., greater the ratio $\frac{L}{r^2}$, greater will be the elongation in the wire.

(iii) Elongation in a wire by its own weight: The weight of the wire Mg act at the center of gravity of the wire so that length of wire which is stretched will be L/2.

$\therefore$ Elongation $l = \frac{FL}{AY} = \frac{Mg(L/2)}{AY} = \frac{MgL}{2AY} = \frac{L^2 dg}{2Y}$ [As mass (M) = volume (AL) × density (d)]

(iv) Thermalstress: If a rod is fixed between two rigid supports, due to change in temperature its length will change and so it will exert a normal stress (compressive if temperature increases and tensile if temperature decreases) on the supports. This stress is called thermal s

As by definition, coefficient of linear expansion $\alpha = \frac{l}{L\Delta\theta}$

⇒ Thermal strain $\frac{l}{L} = \alpha\Delta\theta$

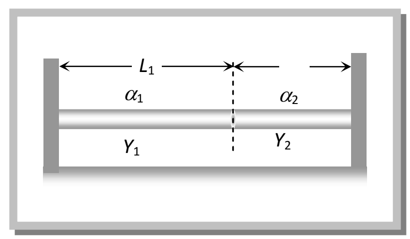
So thermal stress = $Y\alpha\Delta\theta$ [As Y = stress/strain]

And tensile or compressive force produced in the body = $YA\alpha\Delta\theta$

Note: In case of volume expansion Thermal stress = $K\gamma\Delta\theta$

Where K = Bulk modulus, γ = coefficient of cubical expansion

(v) Force between the two rods: Two rods of different metals, having the same area of cross section A, are placed end to end between two massive walls as shown in figure. The first rod has a length L1, coefficient of linear expansion α1 and young's modulus Y1. The corresponding quantities for second rod are L2, α2 and Y2. If the temperature of both the rods is now raised by T degrees.



Increase in length of the composite rod (due to heating) will be equal to

$l_1 + l_2 = [L_1\alpha_1 + L_2\alpha_2]T$ [As l = L $\alpha\Delta\theta$]

And due to compressive force F from the walls due to elasticity,

Decrease in length of the composite rod will be equal to $\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right]\frac{F}{A}$ $\left[\text{As}\, l = \frac{FL}{AY}\right]$

As the length of the composite rod remains unchanged the increase in length due to heating must be equal to decrease in length due to compression i.e. $\frac{F}{A}\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right] = [L_1\alpha_1 + L_2\alpha_2]T$

or $$F = \frac{A[L_1\alpha_1 + L_2\alpha_2]T}{\left[\frac{L_1}{Y_1} + \frac{L_2}{Y_2}\right]}$$

(vi) Force constant of wire: Force required to produce unit elongation in a wire is called force constant of material of wire. It is denoted by k.

$\therefore \quad k = \dfrac{F}{l}$ .....(i)

But from the definition of young's modulus $Y = \dfrac{F/A}{l/L} \Rightarrow \dfrac{F}{l} = \dfrac{YA}{L}$ .....(ii)

From (i) and (ii) $k = \dfrac{YA}{L}$

It is clear that the value of force constant depends upon the dimension (length and area of cross section) and material of a substance.

(vii) Actual length of the wire: If the actual length of the wire is L, then under the tension T1, its length becomes L1 and under the tension T2, its length becomes L2.

$L_1 = L + l_1 \Rightarrow L_1 = L + \dfrac{T_1}{k}$ ...... (i) and $L_2 = L + l_2 \Rightarrow L_2 = L + \dfrac{T_2}{k}$ .....(ii)

From (i) and (ii) we get $L = \dfrac{L_1 T_2 - L_2 T_1}{T_2 - T_1}$

When longitudinal tension 4N is applied on it $L + \dfrac{4}{k} = a$ .....(i)

And when longitudinal tension 5N is applied on it $L + \dfrac{5}{k} = b$ .....(ii)

By solving (i) and (ii) we get $k = \dfrac{1}{b-a}$ and $L = 5a - 4b$

Now when longitudinal tension 9N is applied on elastic string then its length = $L + \dfrac{9}{k}$
$= 5a - 4b + 9(b-a) = 5b - 4a$