Work Done in Stretching a Wire.

In stretching a wire work is done against internal restoring forces. This work is stored in the wire as elastic potential energy or strain energy.

If a force F acts along the length L of the wire of cross-section A and stretches it by x then

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{x/L} = \frac{FL}{Ax} \Rightarrow F = \frac{YA}{L}.x$$

So the work done for an additional small increase dx in length, $dw = Fdx = \frac{YA}{L}x\,.dx$

Hence the total work done in increasing the length by l,

$$W = \int_0^l dW = \int_0^l Fdx = \int_0^l \frac{YA}{L}.x\,dx = \frac{1}{2}\frac{YA}{L}l^2$$

This work done is stored in the wire.

$\therefore$ Energy stored in wire $U = \frac{1}{2}\frac{YAl^2}{L} = \frac{1}{2}Fl$ $\left[\text{As } F = \frac{YAl}{L}\right]$

Dividing both sides by volume of the wire we get energy stored in per unit volume of wire.

$$U_V = \frac{1}{2}\times\frac{F}{A}\times\frac{l}{L} = \frac{1}{2}\times\text{stress}\times\text{strain} = \frac{1}{2}\times Y\times(\text{strain})^2 = \frac{1}{2Y}(\text{stress})^2$$ [As AL = volume of wire]

Total energy stored in wire (U)	Energy stored in per unit volume of wire (UV)
$\frac{1}{2}Fl$	$\frac{1}{2}\frac{Fl}{\text{volume}}$
$\frac{1}{2}\times\text{stress}\times\text{strain}\times\text{volume}$	$\frac{1}{2}\times\text{stress}\times\text{strain}$
$\frac{1}{2}\times Y\times(\text{strain})^2\times\text{volume}$	$\frac{1}{2}\times Y\times(\text{strain})^2$
$\frac{1}{2Y}\times(\text{stress})^2\times\text{volume}$	$\frac{1}{2Y}\times(\text{stress})^2$

Note: If the force on the wire is increased from F1 to F2 and the elongation in wire is l then energy stored in the wire $U = \frac{1}{2}\frac{(F_1 + F_2)}{2}l$

Thermal energy density = Thermal energy per unit volume = $\frac{1}{2}$ × Thermal stress × strain

$= \frac{1}{2}\frac{F}{A}\frac{l}{L}$ $= \frac{1}{2}(Y\alpha\Delta\theta)(\alpha\Delta\theta)$ $= \frac{1}{2}Y\alpha^2(\Delta\theta)^2$