

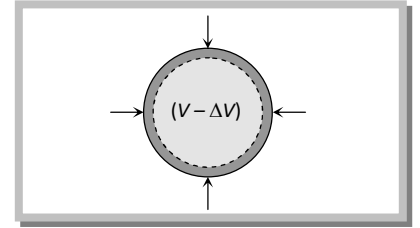
## Bulk Modulus.

When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, such that the shape remains the same, then there is a change in volume.

Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus. This is denoted by K.

$$K = \frac{\text{Normal stress}}{\text{volumetric strain}}$$

$$K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$



Where p = increase in pressure; V = original volume;  $\Delta V$  = change in volume

The negative sign shows that with increase in pressure p, the volume decreases by  $\Delta V$  i.e. if p is positive,  $\Delta V$  is negative. The reciprocal of bulk modulus is called compressibility.

$$C = \text{compressibility} = \frac{1}{K} = \frac{\Delta V}{pV}$$

S.I. unit of compressibility is  $\text{N}^{-1}\text{m}^2$  and C.G.S. unit is  $\text{dyne}^{-1}\text{cm}^2$ .

Gases have two bulk moduli, namely isothermal elasticity  $E_\theta$  and adiabatic elasticity  $E_\phi$ .

(1) Isothermal elasticity ( $E_\theta$ ): Elasticity possess by a gas in isothermal condition is defined as isothermal elasticity.

For isothermal process,  $PV = \text{constant}$  (Boyle's law)

Differentiating both sides  $PdV + VdP = 0 \Rightarrow PdV = -VdP$

$$P = \frac{dP}{(-dV/V)} = \frac{\text{stress}}{\text{strain}} = E_\theta$$

$\therefore E_\theta = P$

i.e., Isothermal elasticity is equal to pressure.

(2) Adiabatic elasticity ( $E_\phi$ ): Elasticity possess by a gas in adiabatic condition is defined as adiabatic elasticity.

For adiabatic process,  $PV^\gamma = \text{constant}$  (Poisson's law)

Differentiating both sides,  $P\gamma V^{\gamma-1}dV + V^\gamma dP = 0 \Rightarrow \gamma PdV + VdP = 0$

$$\gamma P = \frac{dP}{\left(\frac{-dV}{V}\right)} = \frac{\text{stress}}{\text{strain}} = E_{\phi}$$

$$\therefore E_{\phi} = \gamma P$$

i.e., adiabatic elasticity is equal to  $\gamma$  times pressure.

$$[\text{Where } \gamma = \frac{C_p}{C_v}]$$

Note: Ratio of adiabatic to isothermal elasticity  $\frac{E_{\phi}}{E_{\theta}} = \frac{\gamma P}{P} = \gamma > 1 \therefore E_{\phi} > E_{\theta}$

i.e., adiabatic elasticity is always more than isothermal elasticity.