Relation between Volumetric Strain, Lateral Strain and Poisson's ration:

If a long bar have a length L and radius r then volume

$$V=\pi r^2 L$$

Differentiating both the sides

$$dV=\pi r^2 dL+\pi 2rL dr$$

Dividing both the sides by volume of bar

$$\frac{dV}{V}=\frac{\pi r^2 dL}{\pi r^2 L}+\frac{\pi 2rL dr}{\pi r^2 L}$$

$$=\frac{dL}{L}+2\frac{dr}{r}$$

$\Rightarrow$ Volumetric strain = longitudinal strain + 2(lateral strain)

$\Rightarrow \frac{dV}{V}=\frac{dL}{L}+2\sigma\frac{dL}{L} = (1+2\sigma)\frac{dL}{L}$

$\left[As\ \sigma=\frac{dr/r}{dL/L} \Rightarrow \frac{dr}{r}=\sigma\frac{dL}{L}\right]$ or $\sigma=\frac{1}{2}\left[\frac{dV}{A\,dL}-1\right]$

[where A = cross-section of bar]

(i) If a material having

$$\sigma=-0.5$$

then

$$\frac{dV}{V}=[1+2\sigma]\frac{dL}{L}=0$$

∴ Volume = constant or

$$K=\infty$$

i.e. the material is incompressible.

(ii) If a material having $\sigma=0$,

then lateral strain is zero i.e. when a substance is stretched its length increases without any decrease in diameter e.g. cork. In this case change in volume is maximum.

(iii) Theoretical value of Poisson's ratio

$$-1<\sigma<0.5$$

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(iv) Practical value of Poisson's ratio

$0<\sigma<0.5$