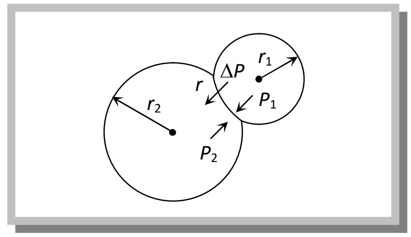
Useful Facts and Formulae.

(1) Formation of double bubble: If r1 and r2 are the radii of smaller and larger bubble and P0 is the atmospheric pressure, then the pressure inside them will be $P_1 = P_0 + \frac{4T}{r_1}$ and $P_2 = P_0 + \frac{4T}{r_2}$.

Now as $r_1 < r_2 \therefore P_1 > P_2$



So for interface $\Delta P = P_1 - P_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ .....(i)

As excess pressure acts from concave to convex side, the interface will be concave towards the smaller bubble and convex towards larger bubble and if r is the radius of interface.

$$\Delta P = \frac{4T}{r} \quad .....(ii)$$

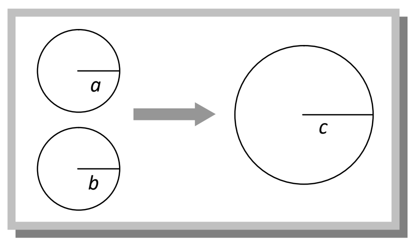
From (i) and (ii) $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$

$\therefore$ Radius of the interface $r = \frac{r_1 r_2}{r_2 - r_1}$

(2) Formation of a single bubble

(i) Under isothermal condition two soap bubble of radii 'a' and 'b' coalesce to form a single bubble of radius 'c'.

If the external pressure is P0 then pressure inside bubbles



$P_a = \left(P_0 + \frac{4T}{a}\right)$, $P_b = \left(P_0 + \frac{4T}{b}\right)$ and $P_c = \left(P_0 + \frac{4T}{c}\right)$

and volume of the bubbles

$V_a = \frac{4}{3}\pi a^3$, $V_b = \frac{4}{3}\pi b^3$, $V_c = \frac{4}{3}\pi c^3$

Now as mass is conserved $\mu_a + \mu_b = \mu_c \Rightarrow \frac{P_a V_a}{RT_a} + \frac{P_b V_b}{RT_b} = \frac{P_c V_c}{RT_c}$ $\left[\text{As } PV = \mu RT, \text{ i.e., } \mu = \frac{PV}{RT}\right]$

$\Rightarrow P_a V_a + P_b V_b = P_c V_c$ .....(i) [As temperature is constant, i.e., $T_a = T_b = T_c$]

Substituting the value of pressure and volume

$$\Rightarrow \left[P_0 + \frac{4T}{a}\right]\left[\frac{4}{3}\pi a^3\right] + \left[P_0 + \frac{4T}{b}\right]\left[\frac{4}{3}\pi b^3\right] = \left[P_0 + \frac{4T}{c}\right]\left[\frac{4}{3}\pi c^3\right]$$

$\Rightarrow \quad 4T(a^2 + b^2 - c^2) = P_0(c^3 - a^3 - b^3)$

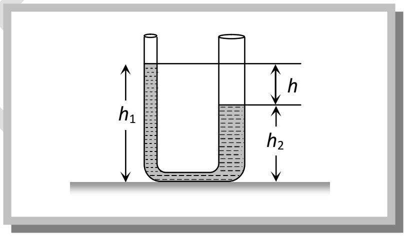
$\therefore$ Surface tension of the liquid $T = \dfrac{P_0(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

(ii) If two bubble coalesce in vacuum then by substituting $P_0 = 0$ in the above expression we get

$$a^2 + b^2 - c^2 = 0 \therefore c^2 = a^2 + b^2$$

Radius of new bubble $= c = \sqrt{a^2 + b^2}$ or can be expressed as $r = \sqrt{r_1^2 + r_2^2}$.

(3) The difference of levels of liquid column in two limbs of u-tube of unequal radii r1 and r2 is



$$h = h_1 - h_2 = \frac{2T\cos\theta}{dg}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

(4) A large force (F) is required to draw apart normally two glass plate enclosing a thin water film because the thin water film formed between the two glass plates will have concave surface all around. Since on the concave side of a liquid surface, pressure is more, work will have to be done in drawing the plates apart.

$F = \dfrac{2AT}{t}$ Where T= surface tension of water film, t= thickness of film, A = area of film.

(5) When a soap bubble is charged, then its size increases due to outward force on the bubble.

(6) The materials, which when coated on a surface and water does not enter through that surface are known as water proofing agents. For example wax etc. Water proofing agent increases the angle of contact.

(7) Values of surface tension of some liquids.

Liquid	Surface tension Newton/meter

Mercury	0.465
Water	0.075
Soap solution	0.030
Glycerin	0.063
Carbon tetrachloride	0.027
Ethyl alcohol	0.022