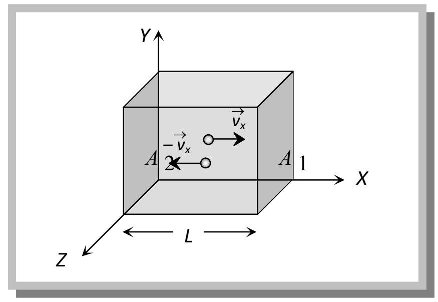
Pressure of an Ideal Gas.

Consider an ideal gas (consisting of N molecules each of mass m) enclosed in a cubical box of side L.

It's any molecule moves with velocity $\vec{v}$ in any direction where

$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$



This molecule collides with the shaded wall (A_1) with velocity v_x and rebounds with velocity $-v_x$.

The change in momentum of the molecule $\Delta P = (-mv_x) - (mv_x) = -2mv_x$

As the momentum remains conserved in a collision, the change in momentum of the wall A1 is $\Delta P = 2mv_x$

After rebound this molecule travel toward opposite wall A2 with velocity $-v_x$, collide to it and again rebound with velocity v_x towards wall A1.

(1) Time between two successive collisions with the wall A1.

$$\Delta t = \frac{\text{Distance travelled by molecule between two successive collision}}{\text{Velocity of molecule}} = \frac{2L}{v_x}$$

$\therefore$ Number of collision per second $n = \frac{1}{\Delta t} = \frac{v_x}{2L}$

(2) The momentum imparted per unit time to the wall by this molecule $n\Delta P = \frac{v_x}{2L} 2mv_x = \frac{m}{L} v_x^2$

This is also equal to the force exerted on the wall A_1 due to this molecule $\therefore \Delta F = \frac{m}{L} v_x^2$

(3) The total force on the wall A_1 due to all the molecules $F_x = \frac{m}{L} \sum v_x^2$

(4) Now pressure is defined as force per unit area

$\therefore$ $P_x = \frac{F_x}{A} = \frac{m}{AL}\sum v_x^2 = \frac{m}{V}\sum v_x^2$ Similarly $P_y = \frac{m}{V}\sum v_y^2$ and $P_z = \frac{m}{V}\sum v_z^2$

So $P_x + P_y + P_z = \frac{m}{V}\sum (v_x^2 + v_y^2 + v_z^2)$

$3P = \frac{m}{V}\sum v^2$ [As $P_x = P_y = P_z = P$ and $v^2 = v_x^2 + v_y^2 + v_z^2$]

$3P = \frac{m}{V}(v_1^2 + v_2^2 + v_3^3 + ...)$

or $3P = \frac{mN}{V}\left(\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + ...}{N}\right)$

or $3P = \frac{m\,N}{V}v_{rms}^2$ [As root mean square velocity of the gas molecule $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + ...}{N}}$]

or $P = \frac{1}{3}\frac{m\,N}{V}v_{rms}^2$

Important points

(i) $P = \frac{1}{3}\frac{m\,N}{V}v_{rms}^2$ or $P \propto \frac{(m\,N)T}{V}$ [As $v_{rms}^2 \propto T$]

(a) If volume and temperature of a gas are constant P $\propto$ mN i.e. Pressure $\propto$ (Mass of gas).

i.e. if mass of gas is increased, number of molecules and hence number of collision per second increases i.e. pressure will increase.

(b) If mass and temperature of a gas are constant. P $\propto$ (1/V), i.e., if volume decreases, number of collisions per second will increase due to lesser effective distance between the walls resulting in greater pressure.

(c) If mass and volume of gas are constant, $P \propto (v_{rms})^2 \propto T$

i.e., if temperature increases, the mean square speed of gas molecules will increase and as gas molecules are moving faster, they will collide with the walls more often with greater momentum resulting in greater pressure.

(ii) $P = \frac{1}{3}\frac{m\,N}{V}v_{rms}^2 = \frac{1}{3}\frac{M}{V}v_{rms}^2$ [As M = mN = Total mass of the gas]

$\therefore$ $P = \frac{1}{3}\rho v_{rms}^2$ $\left[\text{As } \rho = \frac{M}{V}\right]$

(iii) Relation between pressure and kinetic energy

Kinetic energy $= \frac{1}{2}M v_{rms}^2$ $\therefore$ Kinetic energy per unit volume $(E) = \frac{1}{2}\left(\frac{M}{V}\right)v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2$ .....(i)

and we know $P = \frac{1}{3}\rho v_{rms}^2$ .....(ii)

From (i) and (ii), we get $P = \frac{2}{3}E$

i.e. the pressure exerted by an ideal gas is numerically equal to the two third of the mean kinetic energy of translation per unit volume of the gas.

At constant volume and temperature, if the mass of the gas is doubled then pressure will become twice.