Specific Heat in Terms of Degree of Freedom.

We know that kinetic energy of one mole of the gas, having f degrees of freedom can be given by

$$E = \frac{f}{2} RT \qquad .....(i)$$

where T is the temperature of the gas but from the definition of Cv , if dE is a small amount of heat energy required to raise the temperature of 1 gm mole of the gas at constant volume, through a temperature dT then

$$dE = \mu C_v dT = C_v\, dT \text{ or } C_v = \frac{dE}{dT} \qquad [\text{As } \mu = 1] \qquad .....(ii)$$

Putting the value of E from equation (i) we get $C_v = \frac{d}{dT}\left(\frac{f}{2} RT\right) = \frac{f}{2} R$

$$\therefore \quad C_v = \frac{f}{2} R$$

From the Mayer's formula $C_p - C_v = R \Rightarrow C_p = C_v + R = \frac{f}{2} R + R = \left(\frac{f}{2} + 1\right) R$

$$\therefore \quad C_p = \left(\frac{f}{2} + 1\right) R$$

Ratio of Cp and Cv : $\gamma = \frac{C_p}{C_v} = \frac{\left(\frac{f}{2} + 1\right) R}{\frac{f}{2} R} = 1 + \frac{2}{f}$

$$\therefore \quad \gamma = 1 + \frac{2}{f}$$

Important points

(i) Value of γ is always more than 1. So we can say that always Cp > Cv .

(ii) Value of γ is different for monoatomic, diatomic and triatomic gases.

(iii) As $\gamma = 1 + \frac{2}{f} \Rightarrow \frac{2}{f} = \gamma - 1 \Rightarrow \frac{f}{2} = \frac{1}{\gamma - 1}$

$\therefore C_v = \frac{f}{2} R = \frac{R}{\gamma - 1}$

and $C_p = \left(\frac{f}{2} + 1\right) R = \left(\frac{1}{\gamma - 1} + 1\right) R = \left(\frac{\gamma}{\gamma - 1}\right) R$

Specific heat and kinetic energy for different gases

		Monoatomic	Diatomic	Triatomic non-linear	Triatomic linear
Atomicity	A	1	2	3	3
Restriction	B	0	1	3	2
Degree of freedom	f = 3A – B	3	5	6	7
Molar specific heat at constant volume	$C_v = \frac{f}{2} R = \frac{R}{\gamma - 1}$	$\frac{3}{2} R$	$\frac{5}{2} R$	3R	$\frac{7}{2} R$
Molar specific heat at constant pressure	$C_p = \left(\frac{f}{2} + 1\right) R = \left(\frac{\gamma}{\gamma - 1}\right) R$	$\frac{5}{2} R$	$\frac{7}{2} R$	4R	$\frac{9}{2} R$
Ratio of Cp and Cv	$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$	$\frac{5}{3} \simeq 1.66$	$\frac{7}{5} \simeq 1.4$	$\frac{4}{3} \simeq 1.33$	$\frac{9}{7} \simeq 1.28$
Kinetic energy of 1 mole	$E_{\text{mole}} = \frac{f}{2} RT$	$\frac{3}{2} RT$	$\frac{5}{2} RT$	3RT	$\frac{7}{2} RT$

Kinetic energy of 1 molecule	$E_{\text{molecule}} = \frac{f}{2}kT$	$\frac{3}{2}kT$	$\frac{5}{2}kT$	3kT	$\frac{7}{2}kT$
Kinetic energy of 1 gm	$E_{\text{gram}} = \frac{f}{2}rT$	$\frac{3}{2}rT$	$\frac{5}{2}rT$	3rT	$\frac{7}{2}rT$