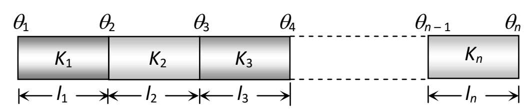
Combination of Conductors.

(1) Series combination:

Let n slabs each of cross-sectional area A, lengths $l_1, l_2, l_3, ... l_n$ and conductivities $K_1, K_2, K_3 ...... K_n$ respectively be connected in the series



Heat current is the same in all the conductors.

i.e. $\frac{Q}{t} = H_1 = H_2 = H_3 .... = H_n$

$$\frac{K_1 A(\theta_1 - \theta_2)}{l_1} = \frac{K_2 A(\theta_2 - \theta_3)}{l_2} = \frac{K_3 A(\theta_3 - \theta_4)}{l_3} = ... = \frac{K_n A(\theta_{n-1} - \theta_n)}{l_n}$$

(i) Equivalent resistance $R = R_1 + R_2 + R_3 + ... R_n$

(ii) If K_s is equivalent conductivity, then from relation $R = \frac{l}{KA}$

$$\frac{l_1 + l_2 + l_3 + ... l_n}{K_s} = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A} + \frac{l_3}{K_3 A} + ... + \frac{l_n}{K_n A}$$

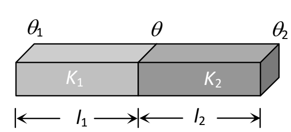
$$\therefore \quad K_s = \frac{l_1 + l_2 + l_3 + ... l_n}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3} + ... \frac{l_n}{K_n}}$$

(iii) Equivalent thermal conductivity for n slabs of equal length $K = \frac{n}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + ... \frac{1}{K_n}}$

For two slabs of equal length, $K = \frac{2K_1 K_2}{K_1 + K_2}$

(iv) Temperature of interface of composite bar: Let the two bars are arranged in series as shown in the figure.

Then heat current is same in the two conductors.



i.e. $\frac{Q}{t} = \frac{K_1 A(\theta_1 - \theta)}{l_1} = \frac{K_2 A(\theta - \theta_2)}{l_2}$

$$\theta = \frac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$$

By solving we get

If $(l_1 = l_2 = l)$ then $\theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2}$

(2) Parallel Combination

Let n slabs each of length l, areas $A_1, A_2, A_3, ... A_n$ and thermal conductivities $K_1, K_2, K_3, ... K_n$ are connected in parallel then.

(i) Equivalent resistance $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \frac{1}{R_n}$

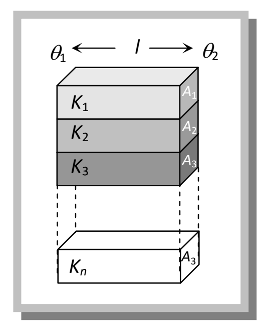
(ii) Temperature gradient across each slab will be same.

(iii) Heat current in each slab will be different. Net heat current will be the sum of heat currents through individual slabs. i.e. $H = H_1 + H_2 + H_3 + ... H_n$

$$\frac{K(A_1 + A_2 + A_3 + ... + A_n)(\theta_1 - \theta_2)}{l}$$

$$= \frac{K_1 A_1 (\theta_1 - \theta_2)}{l} + \frac{K_2 A_2 (\theta_1 - \theta_2)}{l} + \frac{K_3 A_3 (\theta_1 - \theta_2)}{l} .... + \frac{K_n A_n (\theta_1 - \theta_2)}{l}$$

$$\therefore \quad K = \frac{K_1 A_1 + K_2 A_2 + K_3 A_3 + ... K_n A_n}{A_1 + A_2 + A_3 + ... A_n}$$



For n slabs of equal area $K = \frac{K_1 + K_2 + K_3 + ... K_n}{n}$

Equivalent thermal conductivity for two slabs of equal area $K = \frac{K_1 + K_2}{2}$