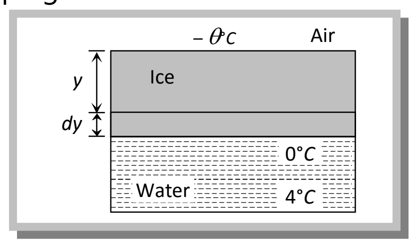
Growth of Ice on Lake.

Water in a lake starts freezing if the atmospheric temperature drops below $0^o C$. Let y be the thickness of ice layer in the lake at any instant t and atmospheric temperature is $-\theta^o C$. The temperature of water in contact with lower surface of ice will be zero. If A is the area of lake, heat escaping through ice in time dt is

$$dQ_1 = \frac{KA[0-(-\theta)]\,dt}{y}$$

Now, suppose the thickness of ice layer increases by dy in time dt, due to escaping of above heat. Then



$$dQ_2 = mL = \rho(dy\ A)L$$

As $dQ_1 = dQ_2$, hence, rate of growth of ice will be $(dy/dt) = (K\theta/\rho L y)$

So, the time taken by ice to grow to a thickness y is $t = \frac{\rho L}{K\theta}\int_0^y y\,dy = \frac{\rho L}{2K\theta}y^2$

If the thickness is increased from y_1 to y_2 then time taken $t = \frac{\rho L}{K\theta}\int_{y_1}^{y_2} y dy = \frac{\rho L}{2K\theta}(y_2^2 - y_1^2)$

(i) Take care and do not apply a negative sign for putting values of temperature in formula and also do not convert it to absolute scale.

(ii) Ice is a poor conductor of heat, therefore the rate of increase of thickness of ice on ponds decreases with time.

(iii) It follows from the above equation that time taken to double and triple the thickness, will be in the ratio of

$$t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2, \text{ i.e., } t_1 : t_2 : t_3 :: 1 : 4 : 9$$

(iv) The time intervals to change the thickness from 0 to y, from y to 2y and so on will be in the ratio

$$\Delta t_1 : \Delta t_2 : \Delta t_3 :: (1^2 - 0^2) : (2^2 - 1^2) : (3^2 : 2^2);\ \Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$$