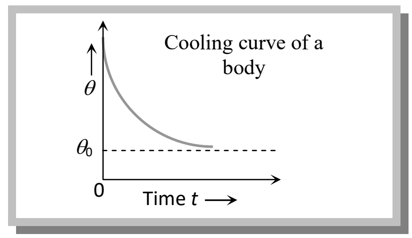
Newton's Law of Cooling.

If in case of cooling by radiation the temperature T of body is not very different from that of surrounding

i.e. $T = T_0 + \Delta T$

$$T^4 - T_0^4 = [(T_0 + \Delta T)^4 - T_0^4] = T_0^4\left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right] = T_0^4\left(1 + \frac{4\Delta T}{T_0} - 1\right)$$ [Using Binomial theorem]

$$= 4T_0^3 \Delta T \qquad .....(i)$$



By Stefan's law, $\frac{dT}{dt} = \frac{eA\sigma}{mc}[T^4 - T_0^4]$

From equation (i), $\frac{dT}{dt} = \frac{eA\sigma}{mc} 4T_0^3 \Delta T$

So $\frac{dT}{dt} \propto \Delta T$ or $\frac{d\theta}{dt} \propto \theta - \theta_0$

i.e., if the temperature of body is not very different from surrounding, rate of cooling is proportional to temperature difference between the body and its surrounding. This law is called Newton's law of cooling.

(1) Practical examples

(i) Hot water loses heat in smaller duration as compared to moderate warm water.

(ii) Adding milk in hot tea reduces the rate of cooling.

(2) Greater the temperature difference between body and its surrounding greater will be the rate of cooling.

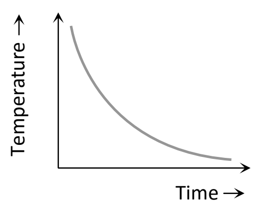
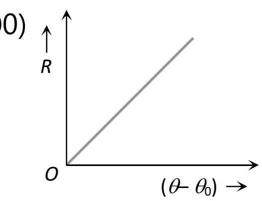
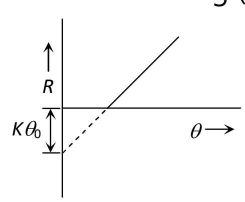
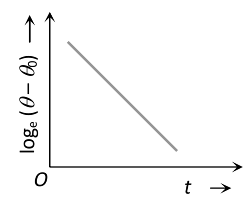
(3) If $\theta = \theta_0$, $\frac{d\theta}{dt} = 0$ i.e. a body can never be cooled to a temperature lesser than its surrounding by radiation.

(4) If a body cools by radiation from $\theta_1^o C$ to $\theta_2^o C$ in time t, then $\frac{d\theta}{dt} = \frac{\theta_1 - \theta_2}{t}$ and $\theta = \theta_{av} = \frac{\theta_1 + \theta_2}{2}$

The Newton's law of cooling becomes $\left[\frac{\theta_1 - \theta_2}{t}\right] = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$

This form of law helps in solving numerical.

(5) Cooling curves:

Curve between temperature of body θ and time.  $\theta - \theta_0 = Ae^{-kt}$, which indicates temperature decreases exponentially with increasing time.	Curve between rate of cooling (R) and temperature difference between body (θ) and surrounding (θ0)  $R \propto (\theta - \theta_0)$. This is a straight line passing through origin.
Curve between the rate of cooling (R) and body temperature (θ).  $R = K(\theta - \theta_0) = K\theta - K\theta_0$ This is a straight line intercept R-axis at $-K\theta_0$	Curve between log(θ – θ0) and time  As $\frac{d\theta}{dt} \propto -(\theta - \theta_0) \Rightarrow \frac{d\theta}{(\theta - \theta_0)} = -Kdt$ Integrating $\log_e(\theta - \theta_0) = -Kt + C$ $\log_e(\theta - \theta_0) = -Kt + \log_e A$ This is a straight line with negative slope

(6) Determination of specific heat of a liquid : If volume, radiating surface area, nature of surface, initial temperature and surrounding of water and given liquid are equal and they are allowed to cool down (by radiation) then rate of loss of heat and fall in temperature of both will be same.

i.e. $$\left(\frac{dQ}{dt}\right)_{\text{water}} = \left(\frac{dQ}{dt}\right)_{\text{liquid}}$$

$$(ms + W)\frac{(\theta_1 - \theta_2)}{t_1} = (m_1 s_1 + W)\frac{(\theta_1 - \theta_2)}{t_2}$$

or $$\left[\frac{ms + W}{t_1}\right] = \left[\frac{m_1 s_1 + W}{t_2}\right]$$ [where W = water equivalent of calorimeter]

If density of water and liquid is ρ and ρ′ respectively then $m = V\rho$ and $m' = V\rho'$