Differential Equation of S.H.M.

For S.H.M. (linear)   Acceleration $\propto$ – (Displacement)

$$A \propto -y$$

or

$$A = -\omega^2 y$$

or

$$\frac{d^2 y}{dt^2} = -\omega^2 y$$

or

$$m\frac{d^2 y}{dt^2} + ky = 0 \qquad\qquad [\text{As } \omega = \sqrt{\frac{k}{m}}\,]$$

For angular S.H.M. $\tau = -c\theta$ and $\dfrac{d^2\theta}{dt^2} + \omega^2\theta = 0$

Where $\omega^2 = \dfrac{c}{I}$ [As c = Restoring torque constant and I = Moment of inertia]