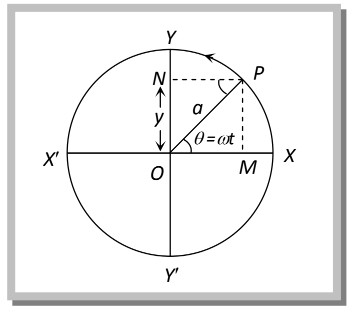
Displacement in S.H.M.

The displacement of a particle executing S.H.M. at an instant is defined as the distance of particle from the mean position at that instant.

As we know that simple harmonic motion is defined as the projection of uniform circular motion on any diameter of circle of reference. If the projection is taken on y-axis.



Then from the figure $y = a\sin\omega t$

$$y = a\sin\frac{2\pi}{T}t$$

$$y = a\sin 2\pi n t$$

$$y = a\sin(\omega t \pm \phi)$$

Where a = Amplitude, ω = Angular frequency, t = Instantaneous time,

T = Time period, n = Frequency and ϕ = Initial phase of particle

If the projection of P is taken on X-axis then equations of S.H.M. can be given as

$$x = a\cos(\omega t \pm \phi)$$

$$x = a\cos\left(\frac{2\pi}{T}t \pm \phi\right)$$

$$x = a\cos(2\pi n t \pm \phi)$$

Important points

(i) $y = a\sin\omega t$ When the time is noted from the instant when the vibrating particle is at mean position.

(ii) $y = a\cos\omega t$ When the time is noted from the instant when the vibrating particle is at extreme position.

(iii) $y = a\sin(\omega t \pm \phi)$ When the vibrating particle is ϕ phase leading or lagging from the mean position.

(iv) Direction of displacement is always away from the equilibrium position, particle either is moving away from or is coming towards the equilibrium position.

(v) If t is given or phase (θ) is given, we can calculate the displacement of the particle.

If $t = \frac{T}{4}$ (or $\theta = \frac{\pi}{2}$) then from equation $y = a\sin\frac{2\pi}{T}t$, we get $y = a\sin\frac{2\pi}{T}\frac{T}{4}$ $= a\sin\left(\frac{\pi}{2}\right) = a$

Similarly if $t = \frac{T}{2}$ (or $\theta = \pi$) then we get $y = 0$