Energy in S.H.M.

A particle executing S.H.M. possesses two types of energy: Potential energy and Kinetic energy

(1) Potential energy: This is an account of the displacement of the particle from its mean position.

The restoring force $F = -ky$ against which work has to be done

So $$U = -\int dw = -\int_0^x F dx = \int_0^y ky\, dy = \frac{1}{2}ky^2$$

$\therefore$ Potential Energy $U = \frac{1}{2}m\omega^2 y^2$ [As $\omega^2 = k/m$]

$U = \frac{1}{2}m\omega^2 a^2 \sin^2 \omega t$ [As $y = a \sin \omega t$]

Important points

(i) Potential energy maximum and equal to total energy at extreme positions

$U_{\max} = \frac{1}{2}ka^2 = \frac{1}{2}m\omega^2 a^2$ When $y = \pm a$; $\omega t = \pi/2$; $t = T/4$

(ii) Potential energy is minimum at mean position

$U_{\min} = 0$ When $y = 0$; $\omega t = 0$ $t = 0$

(2) Kinetic energy: This is because of the velocity of the particle

Kinetic Energy $K = \frac{1}{2}mv^2$

$K = \frac{1}{2}ma^2\omega^2 \cos^2 \omega t$ [As $v = a\omega \cos \omega t$]

$K = \frac{1}{2}m\omega^2(a^2 - y^2)$ [As $v = \omega\sqrt{a^2 - y^2}$]

(i) Kinetic energy is maximum at mean position and equal to total energy at mean position.

$K_{\max} = \frac{1}{2}m\omega^2 a^2$ When $y = 0$; $t = 0$; $\omega t = 0$

(ii) Kinetic energy is minimum at extreme position.

$K_{min} = 0$ When $y = a$; $t = T/4$, $\omega t = \pi/2$

(3) Total energy: Total mechanical energy = Kinetic energy + Potential energy

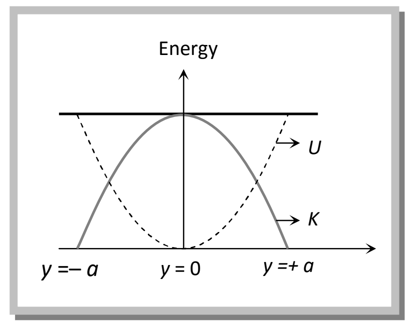
$$E = \frac{1}{2} m\omega^2 (a^2 - y^2) + \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 a^2$$

Total energy is not a position function i.e. it always remains constant.

(4) Energy position graph: Kinetic energy (K) $= \frac{1}{2} m\omega^2 (a^2 - y^2)$

Potential Energy (U) = $\frac{1}{2} m\omega^2 y^2$

Total Energy (E) = $\frac{1}{2} m\omega^2 a^2$



It is clear from the graph that

(i) Kinetic energy is maximum at mean position and minimum at extreme position

(ii) Potential energy is maximum at extreme position and minimum at mean position

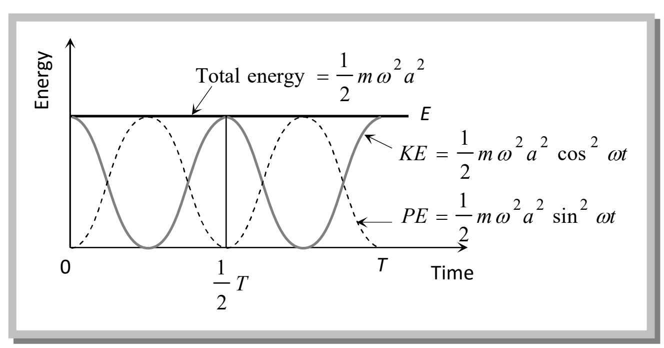
(iii) Total energy always remains constant.

(5) Kinetic Energy $K = \frac{1}{2} m\omega^2 a^2 \cos^2 \omega t = \frac{1}{4} m\omega^2 a^2 (1 + \cos 2\omega t) = \frac{1}{2} E(1 + \cos \omega' t)$

Potential Energy $U = \frac{1}{2} m\omega^2 a^2 \sin^2 \omega t = \frac{1}{4} m\omega^2 a^2 (1 - \cos 2\omega t) = \frac{1}{2} E(1 - \cos \omega' t)$

Where $\omega' = 2\omega$ and $E = \frac{1}{2} m\omega^2 a^2$

i.e. in S.H.M., kinetic energy and potential energy vary periodically with double the frequency of S.H.M. (i.e. with time period $T' = T/2$)



From the graph we note that potential energy or kinetic energy completes two vibrations in a time during which S.H.M. completes one vibration. Thus the frequency of potential energy or kinetic energy double than that of S.H.M.